Static Games of Complete Information-Chapter 1

Nash Equilibrium

Outline of Static Games of Complete Information

- Introduction to games
- Normal-form (or strategic-form) representation
- Iterated elimination of strictly dominated strategies
- Nash equilibrium
- Review of concave functions, optimization
- Applications of Nash equilibrium
- Mixed strategy Nash equilibrium

What is game theory?

- 教材的词云图

What is game theory?

- 我们聚焦于博弈:
  - 有至少两个理性的参与人
  - 每个参与人的选择超过一种
  - 策略外部性: 存在策略互动
  - 结果取决于所有参与人所选择的策略;
- 实例:四个人去饭馆.
  - 每个人为自己的饭付费– 一个简单的决策问题
  - 吃饭前，每个人都同意他们之间平分账单– a game

What is game theory?

- 博弈论是分析一群理性的参与人（或代理人）之间的策略互动的标准方法
- 博弈论的应用
  - 经济学
  - 政治学
  - 社会学
  - 法学
  - 生物学

Classic Example : Prisoners' Dilemma

- 两名犯罪嫌疑人被捕并受到指控，他们被关入不同的牢室。但是警方并无充足证据.
- 两名犯罪嫌疑人被告知以下政策:
  - 如果两人都不坦白，将均被判为轻度犯罪，入狱一个月.
  - 如果双方都坦白，都将被判入狱六个月.
  - 如果一人招认而另一人拒不坦白，招认的一方将马上获释，而另一人将判入狱九个月.

		Prisoner 2: Mum	Prisoner 2: Confess
Prisoner 1	Mum	-1 , -1	-9 , 0
	Confess	0 , -9	-6 , -6

Example : The battle of the sexes

- 在分开的工作场所，Chris 和Pat 必须决定晚上是看歌剧还是去看拳击.
- Chris 和 Pat 都知道以下信息:
  - 两个人都愿意在一起度过这个夜晚.
  - 但是Chris更喜欢歌剧.
  - Pat则更喜欢拳击.

		Pat: Opera	Pat: Prize Fight
Chris	Opera	2 , 1	0 , 0
	Prize Fight	0 , 0	1 , 2

Example : Matching pennies

- 两个参与人都有一枚硬币.
- 两个参与人必须同时选择是Head朝上还是Tail朝上.
- 两个参与人都知道以下规则:
  - 如果两枚硬币配对 (both heads or both tails)， 那么参与人2将赢得参与人1的硬币.
  - 否则, 参与人1会赢得参与人2的硬币.

		Player 2: Head	Player 2: Tail
Player 1	Head	-1 , 1	1 , -1
	Tail	1 , -1	-1 , 1

Static (or simultaneous-move) games of complete information

一个静态（或同时行动）博弈包括的要素:

- 一个参与人集合（至少两个参与人）
- 每个参与人都有一个策略集/行动集
- 每个参与人针对策略组合，或者说对他所偏好的策略组合所获得的收益

- {Player 1, Player 2, ... Player *n*}
- S_1 S_2 ... S_n
- $u_i(s_1, s_2, ...s_n)$, for all $s_1 \in S_1$, $s_2 \in S_2$, ... $s_n \in S_n$.

Static (or simultaneous-move) games of complete information

- 同时行动（Simultaneous-move）
  - 每个参与人在选择他/她的策略时不知道其他参与人的选择.
- 完全信息（Complete information）
  - 每个参与人的策略和收益函数都是所有参与人的共同知识（common knowledge ）.
- 对参与人的假设
  - 理性（Rationality）
    - 参与人的目的是使他的收益最大化
    - 参与人是完美的计算者
  - 每个参与人都知道其他参与人是理性的

Static (or simultaneous-move) games of complete information

- 参与人是否合作?
  - 不.我们仅仅考虑非合作博弈 （non-cooperative games ）
- 时间顺序
  - 每个参与人 i 在不知道其他人的选择的情况下选择他/她的策略s_i.
  - 然后每个参与人 i 得到他/她的收益 $u_i(s_1, s_2, \ldots, s_n)$.
  - 博弈结束.

Definition: normal-form or strategic-form representation

- *一个博弈G的标准式（或策略式）包括:*
  - 一个有限的参与人集合 {1, 2, ..., n},
  - 参与人的策略空间S_1 S_2 ... S_n 和
  - 他们的收益函数u_1 u_2 ... u_n
    其中， $u_i : S_1 \times S_2 \times \ldots \times S_n \rightarrow R$.
  - 我们把这个博弈表示为

$$G = \{S_1, \cdots, S_n; u_1, \cdots u_n\}$$

Normal-form representation: 2-player game

- 双变量矩阵表述
  - 2个参与人: Player 1 和 Player 2
  - 每个参与人有有限数量的策略
- 例如:

$S_1=\{s_{11}, s_{12}, s_{13}\}$ $S_2=\{s_{21}, s_{22}\}$

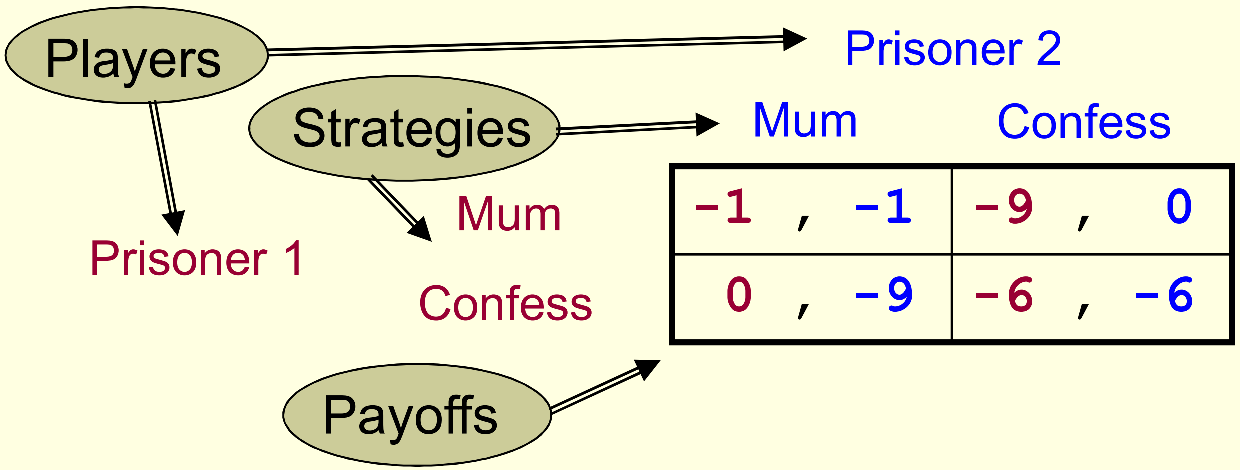
Player 2

		s_{21}	s_{22}
	s_{11}	$u_1(s_{11},s_{21}), u_2(s_{11},s_{21})$	$u_1(s_{11},s_{22}), u_2(s_{11},s_{22})$
Player 1	s_{12}	$u_1(s_{12},s_{21}), u_2(s_{12},s_{21})$	$u_1(s_{12},s_{22}), u_2(s_{12},s_{22})$
	s_{13}	$u_1(s_{13},s_{21}), u_2(s_{13},s_{21})$	$u_1(s_{13},s_{22}), u_2(s_{13},s_{22})$

Classic example: Prisoners' Dilemma: normal-form representation

- 参与人集合: {Prisoner 1, Prisoner 2}
- 策略集: $S_1 = S_2 = \{\underline{\text{M}}\text{um}, \underline{\text{C}}\text{onfess}\}$
- 收益函数:

u_1(M, M)=-1, u_1(M, C)=-9, u_1(C, M)=0, u_1(C, C)=-6;
u_2(M, M)=-1, u_2(M, C)=0, u_2(C, M)=-9, u_2(C, C)=-6



	Mum	Confess
Mum	-1 , -1	-9 , 0
Confess	0 , -9	-6 , -6

Example: The battle of the sexes

		Pat: Opera	Pat: Prize Fight
Chris	Opera	2 , 1	0 , 0
	Prize Fight	0 , 0	1 , 2

- 标准式（或策略式）表述:
  - 参与人集合: { Chris, Pat } (={Player 1, Player 2})
  - 策略集: $S_1 = S_2$ = { Opera, Prize Fight}
  - 收益函数:

    $u_1(O, O)=2$, $u_1(O, F)=0$, $u_1(F, O)=0$, $u_1(F, F)=1$;

    $u_2(O, O)=1$, $u_2(O, F)=0$, $u_2(F, O)=0$, $u_2(F, F)=2$

Example: Matching pennies

		Player 2 Head	Player 2 Tail
Player 1	Head	-1 , 1	1 , -1
	Tail	1 , -1	-1 , 1

- 标准式（或策略式）表述:
  - 参与人集合: {Player 1, Player 2}
  - 策略集: $S_1 = S_2 = \{ \underline{H}ead, \underline{T}ail \}$
  - 收益函数:

    u_1(H, H)=-1, u_1(H, T)=1, u_1(T, H)=1, u_1(T, T)=-1;
    u_2(H, H)=1, u_2(H, T)=-1, u_2(T, H)=-1, u_2(T, T)=1

Example: Tourists & Natives

- 城市里仅有两家酒吧 (bar 1, bar 2)
- 可以索取的价格为$2, $4, or $5
- 6000名游客随机挑选酒吧
- 4000个当地人挑选价格最低的酒吧

- 例1:两家酒吧都索取$2
  - 每家酒吧得到5,000名顾客和 $10,000
- 例2: Bar 1 索取 $4, Bar 2 索取 $5
  - Bar 1 得到3000+4000=7,000名顾客和 $28,000
  - Bar 2 得到3000名顾客和 $15,000

Example: Cournot model of duopoly

- 一种同质的（ homogeneous ）产品仅仅由两家企业进行生产: firm 1 和 firm 2. 产量分别用q_1 和q_2表示. 每家企业选择产量时并不知道其他企业的选择.
- 市场价格是 $P(Q)=a-Q$, 其中$Q=q_1+q_2$.
- firm i 生产产量q_i 的成本是$C_i(q_i)=cq_i$.

标准式表述:

- 参与人集合: **{ Firm 1, Firm 2}**
- 策略集: $S_1=[0, +\infty)$, $S_2=[0, +\infty)$
- 收益函数:

$$u_1(q_1, q_2)=q_1(a-(q_1+q_2)-c),\ \ u_2(q_1, q_2)=q_2(a-(q_1+q_2)-c)$$

One More Example

- n 个参与人同时选择0到100 之间的一个数字. x_i 表示player i 选择的数字.
- y 表示这些数字的平均值
- Player i的收益 = $\boldsymbol{x_i - 3y/5}$
- 标准式表述:
  - 参与人: {player 1, player 2, ..., player n}
  - 策略: $\boldsymbol{S_i}$=[0, 100], for i = 1, 2, ..., n.
  - 收益函数:

$$\boldsymbol{u_i(x_1, x_2, ..., x_n) = x_i - 3y/5}$$

Solving Prisoners' Dilemma

- 无论其他参与人怎样选择，坦白都是更好的策略
- 劣势策略（Dominated strategy）
  - 无论其他参与人怎样选择，都存在比这个策略更好的其他策略
- 例如，在囚徒困境的例子中，无论囚徒2怎样选择，对囚徒1来说，招认（Confess）都是比不招认（Mum）更好的策略。不招认是囚徒1的劣势策略。

		Prisoner 2 Mum	Prisoner 2 Confess
Prisoner 1	Mum	-1 , -1	-9 , 0
	Confess	0 , -9	-6 , -6

Definition: strictly dominated strategy

In the normal-form game $\{S_1, S_2, \ldots, S_n, u_1, u_2, \ldots, u_n\}$, let $s_i', s_i'' \in S_i$ be feasible strategies for player i. Strategy s_i' is ***strictly dominated*** by strategy s_i'' if

$$u_i(s_1, s_2, \ldots s_{i-1}, s_i', s_{i+1}, \ldots, s_n) < u_i(s_1, s_2, \ldots s_{i-1}, s_i'', s_{i+1}, \ldots, s_n)$$

s_i'' is strictly better than s_i'

for all $s_1 \in S_1, s_2 \in S_2, \ldots, s_{i-1} \in S_{i-1}, s_{i+1} \in S_{i+1}, \ldots, s_n \in S_n$.

regardless of other players' choices

Prisoner 2

Prisoner 1	Mum	Confess
Mum	-1 , -1	-9 , 0
Confess	0 , -9	-6 , -6

Example

- 两家企业, Reynolds和Philip, 分享市场
- 如果两家企业都不做广告，则每个企业会从各自顾客那里获得$60百万
- 每个企业的广告成本是$20 百万
- 如果做广告则会从竞争对手那里获得$30 百万

		Philip	
		No Ad	Ad
Reynolds	No Ad	60 , 60	30 , 70
	Ad	70 , 30	40 , 40

2-player game with finite strategies

- $S_1=\{s_{11},\ s_{12},\ s_{13}\}$ $S_2=\{s_{21},\ s_{22}\}$
- s_{11} is strictly dominated by s_{12} if $u_1(s_{11},s_{21})<u_1(s_{12},s_{21})$ and $u_1(s_{11},s_{22})<u_1(s_{12},s_{22})$.
- s_{21} is strictly dominated by s_{22} if $u_2(s_{1i},s_{21}) < u_2(s_{1i},s_{22})$, for $i = 1, 2, 3$

Player 2

Player 1		s_{21}	s_{22}
	s_{11}	$u_1(s_{11},s_{21})$, $u_2(s_{11},s_{21})$	$u_1(s_{11},s_{22})$, $u_2(s_{11},s_{22})$
	s_{12}	$u_1(s_{12},s_{21})$, $u_2(s_{12},s_{21})$	$u_1(s_{12},s_{22})$, $u_2(s_{12},s_{22})$
	s_{13}	$u_1(s_{13},s_{21})$, $u_2(s_{13},s_{21})$	$u_1(s_{13},s_{22})$, $u_2(s_{13},s_{22})$

Definition: weakly dominated strategy

In the normal-form game $\{S_1, S_2, \ldots, S_n, u_1, u_2, \ldots, u_n\}$, let $s_i', s_i'' \in S_i$ be feasible strategies for player i. Strategy s_i' is ***weakly dominated*** by strategy s_i'' if

$$u_i(s_1, s_2, \ldots s_{i-1}, s_i', s_{i+1}, \ldots, s_n)$$
$$\leq (\text{but not always} =)\ u_i(s_1, s_2, \ldots s_{i-1}, s_i'', s_{i+1}, \ldots, s_n)$$

for all $s_1 \in S_1, s_2 \in S_2, \ldots, s_{i-1} \in S_{i-1}, s_{i+1} \in S_{i+1}, \ldots, s_n \in S_n$.

s_i'' is at least as good as s_i'

regardless of other players' choices

Player 2

Player 1

	L	R
U	1 , 1	2 , 0
B	0 , 2	2 , 2

Strictly and weakly dominated strategy

- 一个理性的参与人肯定不会选择严格劣势策略.所以，任何严格劣势策略都可以被剔除.
- 一个理性的参与人有可能选择一个弱劣势策略.
- 剔除的顺序对严格优势策略来说无关紧要，但是对弱优势策略来说就很重要了.

Player 2

		L	C	R
Player 1	T	*2* , *12*	1 , 10	*1* , *12*
	M	0 , 12	1 , 10	0 , 11
	B	0 , 12	0 , 10	0 , 13

- 剔除掉顺序可以是（**B,R,C,M**）和（**C,M,L,B**），结果分别是（**T,L**）和（**T,R**）

Iterated elimination of strictly dominated strategies

- 如果一个策略是严格劣势的，那么剔除它
- 博弈的规模和复杂程度减少（reduced）了
- 在这个简化后的博弈中剔除任何严格劣势策略
- 继续不断的这样做
- 得到理性化的均衡（Rationalizable equilibrium）

Iterated elimination of strictly dominated strategies: an example

Player 2

Player 1

		Left	Middle	Right
Player 1	Up	1 , 0	1 , 2	0 , 1
	Down	0 , 3	0 , 1	2 , 0

Player 2

		Left	Middle
Player 1	Up	1 , 0	1 , 2
	~~Down~~	~~0 , 3~~	~~0 , 1~~

Example: Tourists & Natives

- 城市里仅有两家酒吧 (bar 1, bar 2)
- 可以索取的价格为$2, $4, or $5
- 6000名游客随机挑选酒吧
- 4000个当地人挑选价格最低的酒吧

- 例1:两家酒吧都索取$2
  - 每家酒吧得到5,000名顾客和 $10,000
- 例2: Bar 1 索取 $4, Bar 2 索取 $5
  - Bar 1 得到3000+4000=7,000名顾客和 $28,000
  - Bar 2 得到3000名顾客和 $15,000

Example: Tourists & Natives

		Bar 2		
		$2	$4	$5
Bar 1	~~$2~~	~~10 , 10~~	~~14 , 12~~	~~14 , 15~~
	$4	12 , 14	20 , 20	28 , 15
	$5	15 , 14	15 , 28	25 , 25

Payoffs are in thousands of dollars

		Bar 2	
		$4	$5
Bar 1	$4	20 , 20	28 , 15
	~~$5~~	~~15 , 28~~	~~25 , 25~~

One More Example

- n 个参与人同时选择0到100 之间的一个数字. x_i 表示player i 选择的数字.
- y 表示这些数字的平均值
- Player i的收益 = $\boldsymbol{x_i - 3y/5}$

- 存在劣势策略吗?
- 应该选哪些数字?
- 如果$\boldsymbol{u_i(x_1, x_2, ..., x_n) = x_i - 16y/5}$，会怎么样?

New solution concept: Nash equilibrium

Player 2

		L	C	R
	T	0 , 4	4 , 0	5 , 3
Player 1	M	4 , 0	0 , 4	5 , 3
	B	3 , 5	3 , 5	6 , 6

策略组合(B, R) 有以下性质:

- 如果player 2选R，那么除B以外，Player 1不可能有更好的策略选择.
- 如果player 1选B，那么除 R以外， Player 2不可能有更好的策略选择.

New solution concept: Nash equilibrium

Player 2

		L'	C'	R'
	T'	0 , 4	4 , 0	3 , 3
Player 1	M'	4 , 0	0 , 4	3 , 3
	B'	3 , 3	3 , 3	3.5 , 3.6

策略组合 (B', R') 有以下性质:

- 如果player 2选R'，那么除B'以外，Player 1不可能有更好的策略选择.
- 如果player 1选B'，那么除 R'以外， Player 2不可能有更好的策略选择.

Nash Equilibrium: idea

- 纳什均衡
  - 是一个策略组合。其中，每个参与人选择的策略都是针对其他参与人选择策略的最优反应

Definition: Nash Equilibrium

In the normal-form game $\{S_1, S_2, \ldots, S_n, u_1, u_2, \ldots, u_n\}$, a combination of strategies $(s_1^*, \ldots, s_n^*)$ is a *Nash equilibrium* if, for every player i,

$$u_i(s_1^*, \ldots, s_{i-1}^*, s_i^*, s_{i+1}^*, \ldots, s_n^*) \geq u_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*)$$

Given others' choices, player *i* cannot be better-off if she deviates from s_i^*

(cf: dominated strategy)

for all $s_i \in S_i$. That is, s_i^* solves

Maximize $u_i(s_1^*, \ldots, s_{i-1}^*, s_i, s_{i+1}^*, \ldots, s_n^*)$

Subject to $s_i \in S_i$

Prisoner 2

		Mum	Confess
Prisoner1	Mum	-1 , -1	-9 , 0
	Confess	0 , -9	-6 , -6

2-player game with finite strategies

- $S_1=\{s_{11},\ s_{12},\ s_{13}\}$ $S_2=\{s_{21},\ s_{22}\}$
- $(s_{11},\ s_{21})$ is a Nash equilibrium if
  $u_1(s_{11},s_{21}) \geq u_1(s_{12},s_{21})$,
  $u_1(s_{11},s_{21}) \geq u_1(s_{13},s_{21})$ and
  $u_2(s_{11},s_{21}) \geq u_2(s_{11},s_{22})$.

Player 2

Player 1		s_{21}	s_{22}
	s_{11}	$u_1(s_{11},s_{21})$, $u_2(s_{11},s_{21})$	$u_1(s_{11},s_{22})$, $u_2(s_{11},s_{22})$
	s_{12}	$u_1(s_{12},s_{21})$, $u_2(s_{12},s_{21})$	$u_1(s_{12},s_{22})$, $u_2(s_{12},s_{22})$
	s_{13}	$u_1(s_{13},s_{21})$, $u_2(s_{13},s_{21})$	$u_1(s_{13},s_{22})$, $u_2(s_{13},s_{22})$

Finding a Nash equilibrium: cell-by-cell inspection

		Player 2		
		Left	Middle	Right
Player 1	Up	1 , 0	1 , 2	0 , 1
	Down	0 , 3	0 , 1	2 , 0

Example: Tourists & Natives

		Bar 2		
		$2	$4	$5
	$2	10 , 10	14 , 12	14 , 15
Bar 1	$4	12 , 14	20 , 20	28 , 15
	$5	15 , 14	15 , 28	25 , 25

Payoffs are in thousands of dollars

One More Example

- 标准式表述:
  - 参与人: {player 1, player 2, ..., player n}
  - 策略: $\boldsymbol{S_i}$=[0, 100], for i = 1, 2, ..., n.
  - 收益函数:

$$\boldsymbol{u_i(x_1, x_2, ..., x_n) = x_i - 3y/5}$$

- 哪个策略集是纳什均衡?

Best response function: example

		Player 2 L'	Player 2 C'	Player 2 R'
Player 1	T'	0 , 4	4 , 0	3 , 3
	M'	4 , 0	0 , 4	3 , 3
	B'	3 , 3	3 , 3	3.5 , 3.6

- 如果Player 2 选L'，那么Player 1的最优策略是M'
- 如果Player 2 选C'，那么Player 1的最优策略是T'
- 如果Player 2 选R'，那么Player 1的最优策略是 B'
- 如果Player 1 选B'，那么Player 2的最优策略是 R'

- 最优反应: 给定其他所有参与人的策略，一个参与人能够选择的最优策略

Example: Tourists & Natives

		Bar 2 $2	Bar 2 $4	Bar 2 $5
Bar 1	$2	10 , 10	14 , 12	14 , 15
	$4	12 , 14	20 , 20	28 , 15
	$5	15 , 14	15 , 28	25 , 25

Payoffs are in thousands of dollars

- 针对Bar 2选择的$2, $4 或$5的策略， Bar 1的最优反应分别是什么?
- 针对Bar 1选择的$2, $4 或$5的策略， Bar 2的最优反应分别是什么?

2-player game with finite strategies

- $S_1=\{s_{11}, s_{12}, s_{13}\}$ $S_2=\{s_{21}, s_{22}\}$
- 如果
  $u_1(s_{11},s_{21}) \geq u_1(s_{12},s_{21})$ 且
  $u_1(s_{11},s_{21}) \geq u_1(s_{13},s_{21})$. 那么Player 1的策略
  s_{11} 是她对Player 2策略s_{21}的最优反应，

Player 2

		s_{21}	s_{22}
	s_{11}	$u_1(s_{11},s_{21})$, $u_2(s_{11},s_{21})$	$u_1(s_{11},s_{22})$, $u_2(s_{11},s_{22})$
Player 1	s_{12}	$u_1(s_{12},s_{21})$, $u_2(s_{12},s_{21})$	$u_1(s_{12},s_{22})$, $u_2(s_{12},s_{22})$
	s_{13}	$u_1(s_{13},s_{21})$, $u_2(s_{13},s_{21})$	$u_1(s_{13},s_{22})$, $u_2(s_{13},s_{22})$

Using best response function to find Nash equilibrium

- 在2名参与人的博弈中,当且仅当（i）player 1的策略s_1是对player 2的策略s_2的最优反应,（ii）player 2的策略s_2 是对player 1的策略s_1的最优反应时，(s_1, s_2) 是一个纳什均衡.

		Prisoner 2: Mum	Prisoner 2: Confess
Prisoner 1	Mum	-1 , -1	-9 , <u>0</u>
	Confess	<u>0</u> , -9	<u>-6</u> , <u>-6</u>

Using best response function to find Nash equilibrium: example

Player 2

		L'	C'	R'
Player 1	T'	0 , 4	4 , 0	3 , 3
	M'	4 , 0	0 , 4	3 , 3
	B'	3 , 3	3 , 3	3.5 , 3.6

- M' 是 Player 1对Player 2的策略 L' 的最优反应
- T'是 Player 1对Player 2的策略 C' 的最优反应
- B'是 Player 1对Player 2的策略 R' 的最优反应

- L' 是Player 2对Player 1的策略T'的最优反应
- C'是Player 2对Player 1的策略M'的最优反应
- R'是Player 2对Player 1的策略B' 的最优反应

Example: Tourists & Natives

		Bar 2		
		$2	$4	$5
	$2	10 , 10	14 , 12	14 , 15
Bar 1	$4	12 , 14	20 , 20	28 , 15
	$5	15 , 14	15 , 28	25 , 25

Payoffs are in thousands of dollars

使用最优反应函数找到纳什均衡.

Example: The battle of the sexes

		Pat: Opera	Pat: Prize Fight
Chris	Opera	2 , 1	0 , 0
	Prize Fight	0 , 0	1 , 2

- Opera是Player 1对Player 2的策略Opera的最优反应
- Opera是Player 2对Player 1的策略Opera的最优反应
  - 所以, (Opera, Opera) 是一个纳什均衡

- Fight是Player 1对Player 2的策略Fight的最优反应
- Fight是Player 2对Player 1的策略Fight的最优反应
  - 所以, (Fight, Fight)是一个纳什均衡

Example: Matching pennies

		Player 2	
		Head	Tail
Player 1	Head	-1 , 1	1 , -1
	Tail	1 , -1	-1 , 1

- Head是Player 1对Player 2的策略Tail的最优反应
- Head是Player 2对Player 1的策略Head 的最优反应

- Tail是Player 1对Player 2的策略Head 的最优反应
- Tail是Player 2对Player 1的策略Tail的最优反应
  - 所以, *没有*纯策略纳什均衡

Definition: best response function

In the normal-form game

$$\{S_1, S_2, \ldots, S_n, u_1, u_2, \ldots, u_n\},$$

if player 1, 2, ..., *i*-1, *i*+1, ..., *n* choose strategies $s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n$, respectively,

then player *i*'s best response function is defined by

$$B_i(s_1, \ldots, s_{i-1}, s_{i+1}, \ldots, s_n) =$$

$$\{s_i \in S_i : u_i(s_1, \ldots, s_{i-1}, s_i, s_{i+1}, \ldots, s_n) \geq u_i(s_1, \ldots, s_{i-1}, s_i', s_{i+1}, \ldots, s_n), \text{ for all } s_i' \in S_i\}$$

Given the strategies chosen by other players

Player *i*'s best response

Definition: best response function

An alternative definition:

Player *i*'s strategy $s_i \in B_i(s_1,\ldots,s_{i-1},s_{i+1},\ldots s_n)$ if and only if it solves (or it is an optimal solution to)

Maximize $u_i(s_1,\ldots,s_{i-1},s_i',s_{i+1},\ldots,s_n)$

Subject to $s_i' \in S_i$

where $s_1, \ldots, s_{i-1},\ s_{i+1}, \ldots,\ s_n$ are given.

Player *i*'s best response to other players' strategies is an optimal solution to

Using best response function to define Nash equilibrium

In the normal-form game $\{S_1, \ldots, S_n, u_1, \ldots, u_n\}$, a combination of strategies $(s_1^*, \ldots, s_n^*)$ is a *Nash equilibrium* if for every player *i*,

$$s_i^* \in B_i(s_1^*, \ldots, s_{i-1}^*, s_{i+1}^*, \ldots, s_n^*)$$

- A set of strategies, one for each player, such that each player's strategy is best for her, given that all other players are playing their strategies, or
- A stable situation that no player would like to deviate if others stick to it

Strictly dominated strategies vs. Nash Equilibrium

- 纳什均衡和重复剔除严格劣势策略之间的关系
  - 纳什均衡是一个比重复剔除严格劣势策略更强的解的概念.
  - 可预测性（Predictability）
  - 存在性（Existence）
  - 惟一性（uniqueness）.
  - 如果博弈存在惟一解，它一定是一个纳什均衡

Summary(Appendix 1.1.C)

- *命题A* 在n个参与人的标准式博弈 $G=\{S_1, \ldots, S_n; u_1, \ldots u_n\}$中，如果重复剔除严格劣势策略剔除掉除策略组合$(s_1^*, s_2^*, \ldots, s_n^*)$外的所有策略,那么这一策略组合为该博弈惟一的纳什均衡.
- *命题B* 在n个参与人的标准式博弈 $G=\{S_1, \ldots, S_n; u_1, \ldots u_n\}$中，如果策略$(s_1^*, s_2^*, \ldots, s_n^*)$是一个纳什均衡，那么它不会被重复剔除严格劣势策略所剔除.
- 但是未被重复剔除严格劣势策略所剔除的策略不一定是纳什均衡策略.