1.3 Mixed Strategy Equilibrium

- 1.3.A 混合策略
- 1.3.B 纳什均衡的存在性

- 我们将着重学习：(a) 混合策略的概念；在两个参与人的博弈中 (b) 寻找参与人i对参与人j的混合策略的最优反应；(c) 一个混合策略的策略组合是纳什均衡的充要条件——每个参与者的混合策略是另一个参与者的混合策略的最优反应。(d)讨论混合策略在寻找严格劣势策略中所起的作用。

Matching pennies

		Player 2 Head	Player 2 Tail
Player 1	Head	-1 , 1	1 , -1
	Tail	1 , -1	-1 , 1

- Head是Player 1对Player 2的策略Tail的最优反应
- Tail是Player 2对Player 1的策略Tail的最优反应

- Tail是Player 1对Player 2的策略Head的最优反应
- Head是Player 2对Player 1的策略Head 的最优反应

> 所以, *不存在*（纯策略）纳什均衡

Solving matching pennies

		Player 2	
		Head	**Tail**	
Player 1	**Head**	-1 , 1	1 , -1	*r*
	Tail	1 , -1	-1 , 1	*1-r*
		q	*1-q*	

- 把你的策略随机化会使你的竞争对手感到吃惊
  - Player 1分别以概率r和1-r选择Head和Tail.
  - Player 2分别以概率q和1-q选择Head和Tail.

Solving matching pennies

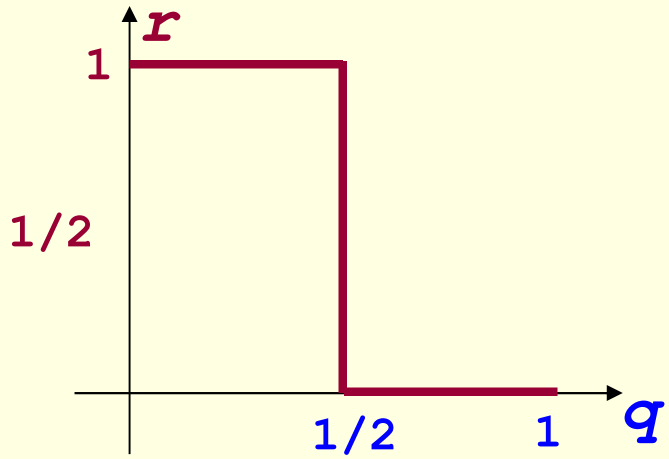
		Player 2			
		Head	**Tail**		Expected payoffs
Player 1	**Head**	-1 , <u>1</u>	<u>1</u> , -1	r	$1-2q$
	Tail	<u>1</u> , -1	-1 , <u>1</u>	$1-r$	$2q-1$
		q	$1-q$		

- Player 1的期望收益
  - 如果Player 1选择Head, $-q+(1-q)=1-2q$
  - 如果Player 1选择Tail, $q-(1-q)=2q-1$

Solving matching pennies

Player 1 \ Player 2	Head	Tail		Expected payoffs
Head	-1 , <u>1</u>	<u>1</u> , -1	r	$1-2q$
Tail	<u>1</u> , -1	-1 , <u>1</u>	$1-r$	$2q-1$
	q	$1-q$		

- Player 1的最优反应
  $B_1(q)$:
  - For $q<0.5$, Head ($r=1$)
  - For $q>0.5$, Tail ($r=0$)
  - For $q=0.5$, indifferent ($0 \le r \le 1$)



Solving matching pennies

		Player 2: Head	Player 2: Tail		Expected payoffs
Player 1	Head	-1 , 1	1 , -1	r	$1-2q$
	Tail	1 , -1	-1 , 1	$1-r$	$2q-1$
		q	$1-q$		
Expected payoffs		$2r-1$	$1-2r$		

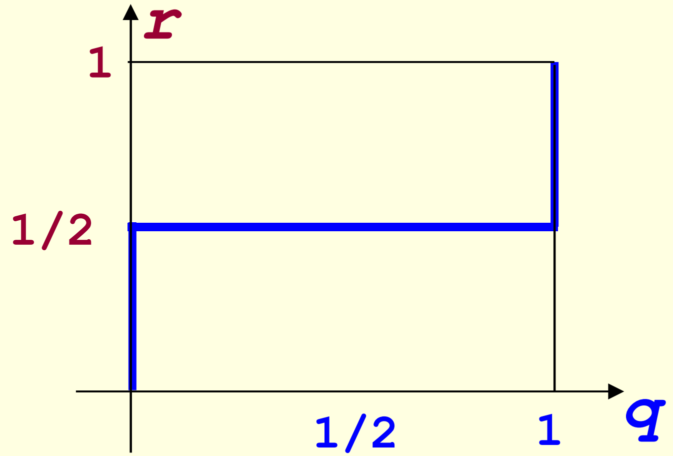
- Player 2的期望收益
  - 如果Player 2选择Head, $r-(1-r)=2r-1$
  - 如果Player 2选择Tail, $-r+(1-r)=1-2r$

Solving matching pennies

		Player 2 Head	Player 2 Tail		Expected payoffs
Player 1	Head	-1 , 1	1 , -1	r	$1-2q$
	Tail	1 , -1	-1 , 1	$1-r$	$2q-1$
		q	$1-q$		
Expected payoffs		$2r-1$	$1-2r$		

- Player 2的最优反应

  $B_2(r)$:
  - For $r<0.5$, Tail ($q=0$)
  - For $r>0.5$, Head ($q=1$)
  - For $r=0.5$, indifferent ($0 \le q \le 1$)



Solving matching pennies

- Player 1的最优反应
  $B_1(q)$：
  - For $q<0.5$, Head $(r=1)$
  - For $q>0.5$, Tail $(r=0)$
  - For $q=0.5$, indifferent $(0\le r\le 1)$
- Player 2的最优反应
  $B_2(r)$：
  - For $r<0.5$, Tail $(q=0)$
  - For $r>0.5$, Head $(q=1)$
  - For $r=0.5$, indifferent $(0\le q\le 1)$

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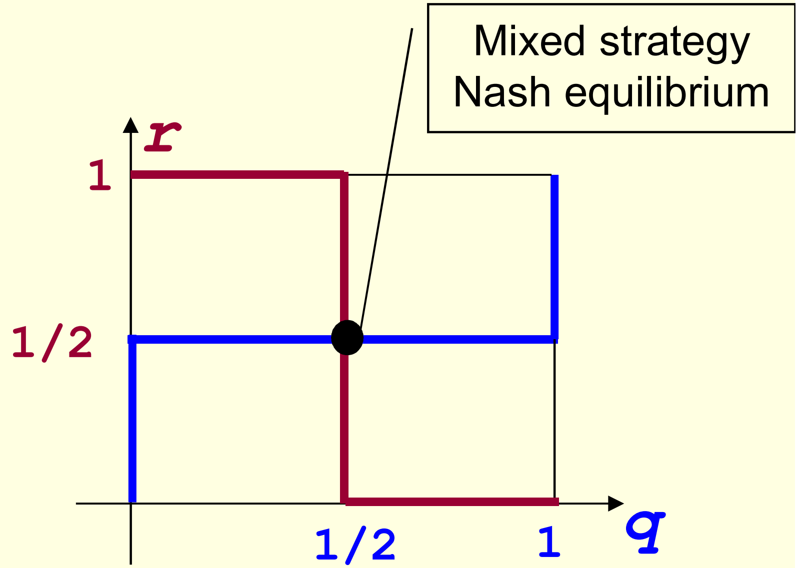
$r = 0.5 \in B_1(0.5)$

$q = 0.5 \in B_2(0.5)$

Player 2

Player 1	Head	Tail	
Head	-1 , 1	1 , -1	r
Tail	1 , -1	-1 , 1	$1-r$
	q	$1-q$	



Mixed strategy

- 混合策略:
  - 一个参与人的混合策略是在参与人的（纯）策略上的概率分布.

Definition Let G be a n-player game with strategy sets S_1, S_2 ,..., S_n. A mixed strategy σ_i for player i is a probability distribution on S_i. If S_i has a finite number of pure strategies, i.e. $S_i = \{s_{i1}, s_{i2}, \ldots s_{iK_i}\}$ then a mixed strategy is a function $\sigma_i : S_i \rightarrow \Re^+$ such that $\sum_{j=1}^{K_i} \sigma_i(s_{ij}) = 1$. We write this mixed strategy as $(\sigma_i(s_{i1}), \sigma_i(s_{i2}), \ldots, \sigma_i(s_{iK_i}))$.

Mixed strategy: example

- 硬币配对(matching pennies)

  - Player 1 有两个纯策略: H和T

    (σ_1(H)=0.5, σ_1(T)=0.5) 是一个混合策略.
    即, player 1分别以0.5和0.5的概率选H和T.

    (σ_1(H)=0.3, σ_1(T)=0.7) 是另一个混合策略.
    即, player 1分别以0.3和0.7的概率选H和T.

Mixed strategy and pure strategy

- 参与人的混合策略是在参与人（纯）策略上的概率分布.
  - Chris的一个混合策略是概率分布(r, 1-r), 其中r 是选择Opera的概率, 1-r 是选择Prize Fight 的概率.
  - 如果r =1，那么 Chris实际上选择了Opera. 如果r = 0，那么Chris实际上选择了Prize Fight.

Battle of sexes

		Pat: Opera	Pat: Prize Fight
Chris	Opera (r)	2 , 1	0 , 0
	Prize Fight (1-r)	0 , 0	1 , 2

Battle of sexes

		Pat	
		Opera (q)	Prize Fight ($1-q$)
Chris	Opera (r)	$\underline{2}$, $\underline{1}$	0 , 0
	Prize Fight ($1-r$)	0 , 0	$\underline{1}$, $\underline{2}$

- Chris选Opera的预期收益: $2q$
- Chris选Prize Fight的预期收益: $1-q$
- Chris的最优反应$B_1(q)$:
  - Prize Fight ($r=0$) if $q<1/3$
  - Opera ($r=1$) if $q>1/3$
  - Any mixed strategy ($0\le r\le 1$) if $q=1/3$

Battle of sexes

		Pat	
		Opera (q)	Prize Fight ($1-q$)
Chris	Opera (r)	2 , 1	0 , 0
	Prize Fight ($1-r$)	0 , 0	1 , 2

- Pat选择Opera的预期收益: r
- Pat选择Prize Fight的预期收益: $2(1-r)$
- Pat的最优反应$B_2(r)$:
  - Prize Fight ($q=0$) if $r<2/3$
  - Opera ($q=1$) if $r>2/3$
  - Any mixed strategy ($0\leq q\leq 1$) if $r=2/3$,

Battle of sexes

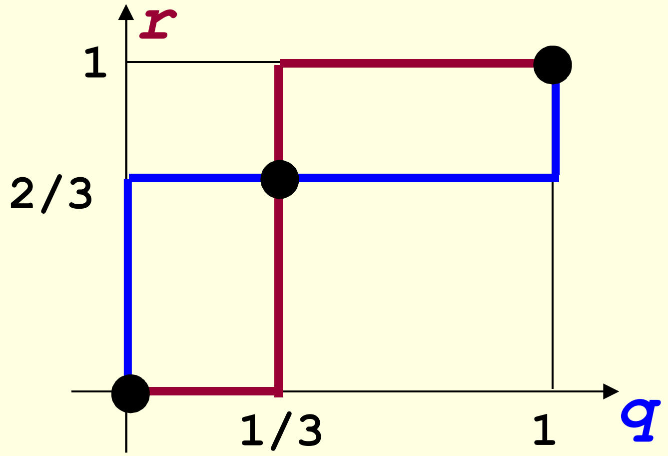
- Chris的最优反应 $B_1(q)$:
  - Prize Fight (r=0) if q<1/3
  - Opera (r=1) if q>1/3
  - Any mixed strategy (0≤r≤1) if q=1/3
- Pat的最优反应$B_2(r)$:
  - Prize Fight (q=0) if r<2/3
  - Opera (q=1) if r>2/3
  - Any mixed strategy (0≤q≤1) if r=2/3

三个纳什均衡:

((1, 0), (1, 0))

((0, 1), (0, 1))

((2/3, 1/3), (1/3, 2/3))



Employee Monitoring

- 雇员可以努力工作也可以偷懒卸责
  - 薪水: $100K除非消极怠工被抓
  - 努力工作的成本: $50K
- 经理可以监督也可以不监督
  - 雇员产出的价值: $200K
  - 雇员不工作时的利润: $0
  - 监督的成本: $10K

Employee Monitoring

		Manager: Monitor (q)	Manager: Not Monitor ($1-q$)	Expected payoffs
Employee	Work (r)	50 , 90	50 , 100	50
	Shirk ($1-r$)	0 , -10	100 , -100	$100(1-q)$
	Expected payoffs	$100r-10$	$200r-100$	

- 雇员的最优反应 $B_1(q)$:
  - Shirk (r=0) if $q<0.5$
  - Work (r=1) if $q>0.5$
  - Any mixed strategy ($0\leq r\leq 1$) if $q=0.5$

Employee Monitoring

		Manager		
		Monitor (q)	Not Monitor (1-q)	Expected payoffs
Employee	Work (r)	50 , 90	50 , 100	50
	Shirk (1-r)	0 , -10	100 , -100	100(1-q)
	Expected payoffs	100r-10	200r-100	

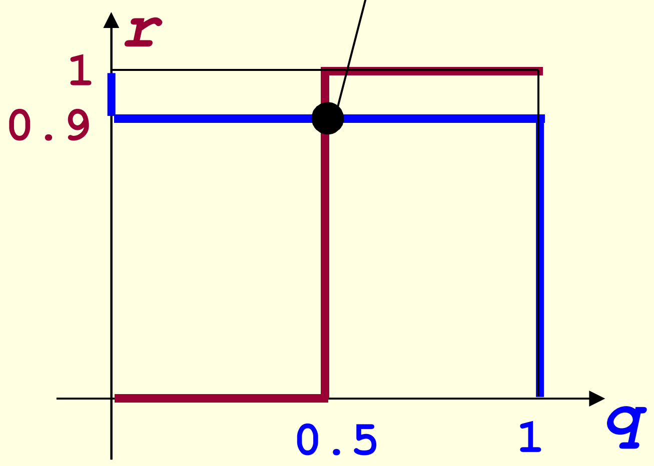
- 经理的最优反应$B_2(r)$：
  - Monitor (q=1) if r<0.9
  - Not Monitor (q=0) if r>0.9
  - Any mixed strategy ($0 \le q \le 1$) if r=0.9

Employee Monitoring

- 雇员的最优反应$B_1(q)$:
  - Shirk ($r=0$) if $q<0.5$
  - Work ($r=1$) if $q>0.5$
  - Any mixed strategy ($0\le r\le 1$) if $q=0.5$
- 经理的最优反应$B_2(r)$:
  - Monitor ($q=1$) if $r<0.9$
  - Not Monitor ($q=0$) if $r>0.9$
  - Any mixed strategy ($0\le q\le 1$) if $r=0.9$

Mixed strategy Nash equilibrium

((0.9,0.1),(0.5,0.5))



Expected payoffs: 2 players each with two pure strategies

		Player 2 s_{21} (q)	Player 2 s_{22} $(1-q)$
Player 1	s_{11} (r)	$u_1(s_{11}, s_{21}), u_2(s_{11}, s_{21})$	$u_1(s_{11}, s_{22}), u_2(s_{11}, s_{22})$
	s_{12} $(1-r)$	$u_1(s_{12}, s_{21}), u_2(s_{12}, s_{21})$	$u_1(s_{12}, s_{22}), u_2(s_{12}, s_{22})$

- Player 1有混合策略 $(r, 1-r)$. Player 2有混合策略 $(q, 1-q)$.
  - Player 1选择s_{11}的期望收益:

    $EU_1(s_{11}, (q, 1-q)) = q \times u_1(s_{11}, s_{21}) + (1-q) \times u_1(s_{11}, s_{22})$
  - Player 1选择s_{12}的期望收益:

    $EU_1(s_{12}, (q, 1-q)) = q \times u_1(s_{12}, s_{21}) + (1-q) \times u_1(s_{12}, s_{22})$
- Player 1混合策略的期望收益:

  $v_1((r, 1-r), (q, 1-q)) = r \times EU_1(s_{11}, (q, 1-q)) + (1-r) \times EU_1(s_{12}, (q, 1-q))$

Expected payoffs: 2 players each with two pure strategies

Player 2

Player 1		s_{21} (q)	s_{22} (1- q)
	s_{11} (r)	$u_1(s_{11}, s_{21})$, $u_2(s_{11}, s_{21})$	$u_1(s_{11}, s_{22})$, $u_2(s_{11}, s_{22})$
	s_{12} (1- r)	$u_1(s_{12}, s_{21})$, $u_2(s_{12}, s_{21})$	$u_1(s_{12}, s_{22})$, $u_2(s_{12}, s_{22})$

- Player 1有混合策略 **(r, 1- r).** Player 2有混合策略 **(q, 1- q).**
  - Player 2选择s_{21}的期望收益:
    $EU_2(s_{21}, (r, 1\text{-}r)) = r \times u_2(s_{11}, s_{21}) + (1\text{-}r) \times u_2(s_{12}, s_{21})$
  - Player 2选择s_{22}的期望收益:
    $EU_2(s_{22}, (r, 1\text{-}r)) = r \times u_2(s_{11}, s_{22}) + (1\text{-}r) \times u_2(s_{12}, s_{22})$
- Player 2混合策略的期望收益:
  $v_2((r, 1\text{-}r),(q, 1\text{-}q)) = q \times EU_2(s_{21}, (r, 1\text{-}r)) + (1\text{-}q) \times EU_2(s_{22}, (r, 1\text{-}r))$

Mixed strategy equilibrium: 2-player each with two pure strategies

Player 2

		s_{21} (q)	s_{22} (1- q)
Player 1	s_{11} (r)	$u_1(s_{11}, s_{21})$, $u_2(s_{11}, s_{21})$	$u_1(s_{11}, s_{22})$, $u_2(s_{11}, s_{22})$
	s_{12} (1- r)	$u_1(s_{12}, s_{21})$, $u_2(s_{12}, s_{21})$	$u_1(s_{12}, s_{22})$, $u_2(s_{12}, s_{22})$

- 混合策略纳什均衡:
  - 一个混合策略组合

    $$((r^*, 1\text{-}r^*), (q^*, 1\text{-}q^*))$$

    是一个纳什均衡，如果 $(r^*,1\text{-}r^*)$是对$(q^*, 1\text{-}q^*)$ 的最优反应, 而$(q^*, 1\text{-}q^*)$是对$(r^*,1\text{-}r^*)$ 的最优反应. 即,

    $v_1((r^*, 1\text{-}r^*), (q^*, 1\text{-}q^*)) \geq v_1((r, 1\text{-}r), (q^*, 1\text{-}q^*))$, for all $0\leq r \leq 1$

    $v_2((r^*, 1\text{-}r^*), (q^*, 1\text{-}q^*)) \geq v_2((r^*, 1\text{-}r^*), (q, 1\text{-}q))$, for all $0\leq q \leq 1$

2-player each with two strategies

		Player 2: s_{21} (q)	Player 2: s_{22} $(1-q)$
Player 1	s_{11} (r)	$u_1(s_{11}, s_{21})$, $u_2(s_{11}, s_{21})$	$u_1(s_{11}, s_{22})$, $u_2(s_{11}, s_{22})$
	s_{12} $(1-r)$	$u_1(s_{12}, s_{21})$, $u_2(s_{12}, s_{21})$	$u_1(s_{12}, s_{22})$, $u_2(s_{12}, s_{22})$

- 定理 1 (混合纳什均衡的性质)
  - 一个混合策略组合 $((r^*, 1-r^*), (q^*, 1-q^*))$ 是一个纳什均衡*当且仅当*

$$v_1((r^*, 1-r^*), (q^*, 1-q^*)) \geq EU_1(s_{11}, (q^*, 1-q^*))$$
$$v_1((r^*, 1-r^*), (q^*, 1-q^*)) \geq EU_1(s_{12}, (q^*, 1-q^*))$$
$$v_2((r^*, 1-r^*), (q^*, 1-q^*)) \geq EU_2(s_{21}, (r^*, 1-r^*))$$
$$v_2((r^*, 1-r^*), (q^*, 1-q^*)) \geq EU_2(s_{22}, (r^*, 1-r^*))$$

Theorem 1: illustration

Matching pennies

		Player 2 H (0.5)	Player 2 T (0.5)
Player 1	H (0.5)	-1 , 1	1 , -1
	T (0.5)	1 , -1	-1 , 1

- Player 1:
  - $EU_1(H, (0.5, 0.5)) = 0.5 \times (-1) + 0.5 \times 1 = 0$
  - $EU_1(T, (0.5, 0.5)) = 0.5 \times 1 + 0.5 \times (-1) = 0$
  - $v_1((0.5, 0.5), (0.5, 0.5)) = 0.5 \times 0 + 0.5 \times 0 = 0$
- Player 2:
  - $EU_2(H, (0.5, 0.5)) = 0.5 \times 1 + 0.5 \times (-1) = 0$
  - $EU_2(T, (0.5, 0.5)) = 0.5 \times (-1) + 0.5 \times 1 = 0$
  - $v_2((0.5, 0.5), (0.5, 0.5)) = 0.5 \times 0 + 0.5 \times 0 = 0$

Theorem 1: illustration

Matching pennies

		Player 2	
		H (0.5)	T (0.5)
Player 1	H (0.5)	-1 , 1	1 , -1
	T (0.5)	1 , -1	-1 , 1

- Player 1:
  - $v_1((0.5, 0.5), (0.5, 0.5)) \geq EU_1(H, (0.5, 0.5))$
  - $v_1((0.5, 0.5), (0.5, 0.5)) \geq EU_1(T, (0.5, 0.5))$
- Player 2:
  - $v_2((0.5, 0.5), (0.5, 0.5)) \geq EU_2(H, (0.5, 0.5))$
  - $v_2((0.5, 0.5), (0.5, 0.5)) \geq EU_2(T, (0.5, 0.5))$
- 所以, 根据定理1，((0.5, 0.5), (0.5, 0.5)) 是一个混合策略纳什均衡.

Theorem 1: illustration

Employee Monitoring

		Manager	
		Monitor (0.5)	Not Monitor (0.5)
Employee	Work (0.9)	50 , 90	50 , 100
	Shirk (0.1)	0 , -10	100 , -100

- Employee选择“work”的期望收益
  - $EU_1(\text{Work}, (0.5, 0.5)) = 0.5 \times 50 + 0.5 \times 50 = 50$
- Employee选择“shirk”的期望收益
  - $EU_1(\text{Shirk}, (0.5, 0.5)) = 0.5 \times 0 + 0.5 \times 100 = 50$
- Employee混合策略的期望收益
  - $v_1((0.9, 0.1), (0.5, 0.5)) = 0.9 \times 50 + 0.1 \times 50 = 50$

Theorem 1: illustration

Employee Monitoring

		Manager	
		Monitor (0.5)	Not Monitor (0.5)
Employee	Work (0.9)	50 , 90	50 , 100
	Shirk (0.1)	0 , -10	100 , -100

- Manager选择"Monitor"的预期收益
  - EU_2(Monitor, (0.9, 0.1)) = 0.9×90+0.1×(-10) =80
- Manager选择"Not"的预期收益
  - EU_2(Not, (0.9, 0.1)) = 0.9×100+0.1×(-100) = 80
- Manager混合策略的预期收益
  - v_2((0.9, 0.1), (0.5, 0.5))=0.5×80+0.5×80=80

Theorem 1: illustration

Employee Monitoring

		Manager	
		Monitor (0.5)	No Monitor (0.5)
Employee	Work (0.9)	50 , 90	50 , 100
	Shirk (0.1)	0 , -10	100 , -100

- Employee
  - $v_1((0.9, 0.1), (0.5, 0.5)) \geq EU_1(\text{Work}, (0.5, 0.5))$
  - $v_1((0.9, 0.1), (0.5, 0.5)) \geq EU_1(\text{Shirk}, (0.5, 0.5))$
- Manager
  - $v_2((0.9, 0.1), (0.5, 0.5)) \geq EU_2(\text{Monitor}, (0.9, 0.1))$
  - $v_2((0.9, 0.1), (0.5, 0.5)) \geq EU_2(\text{Not}, (0.9, 0.1))$
- 所以, 根据定理1，((0.9, 0.1), (0.5, 0.5)) 是一个混合策略纳什均衡.

Theorem 1: illustration

Battle of sexes

		Pat: Opera (1/3)	Pat: Prize Fight (2/3)
Chris	Opera (2/3)	2 , 1	0 , 0
	Prize Fight (1/3)	0 , 0	1 , 2

- 使用命题1检查((2/3, 1/3), (1/3, 2/3))是否是一个混合策略纳什均衡.

Mixed strategy equilibrium: 2-player each with two strategies

		Player 2	
		s_{21} (q)	s_{22} (1- q)
Player 1	s_{11} (r)	$u_1(s_{11}, s_{21})$, $u_2(s_{11}, s_{21})$	$u_1(s_{11}, s_{22})$, $u_2(s_{11}, s_{22})$
	s_{12} (1- r)	$u_1(s_{12}, s_{21})$, $u_2(s_{12}, s_{21})$	$u_1(s_{12}, s_{22})$, $u_2(s_{12}, s_{22})$

- 定理 2　令$((r^*, 1\text{-}r^*), (q^*, 1\text{-}q^*))$是一个混合策略组合, 其中 $0<r^*<1$, $0<q^*<1$. 那么 $((r^*, 1\text{-}r^*), (q^*, 1\text{-}q^*))$是一个混合策略纳什均衡，当且仅当

$$EU_1(s_{11}, (q^*, 1\text{-}q^*)) = EU_1(s_{12}, (q^*, 1\text{-}q^*))$$
$$EU_2(s_{21}, (r^*, 1\text{-}r^*)) = EU_2(s_{22}, (r^*, 1\text{-}r^*))$$

- 即,对于每个参与人来说，她的两个策略都是无差异的.

Use Theorem 2 to find mixed strategy Nash equilibrium: illustration

Matching pennies

		Player 2	
		H (q)	T ($1-q$)
Player 1	H (r)	-1 , 1	1 , -1
	T ($1-r$)	1 , -1	-1 , 1

- Player 1选择 Head 和Tail无差异.
  - $EU_1(H, (q, 1-q)) = q \times (-1) + (1-q) \times 1 = 1-2q$
  - $EU_1(T, (q, 1-q)) = q \times 1 + \times (1-q)\ (-1) = 2q-1$

  - $EU_1(H, (q, 1-q)) = EU_1(T, (q, 1-q))$
    $1-2q = 2q-1$
    $4q = 2$ 从而 $q = 1/2$

Use Theorem 2 to find mixed strategy Nash equilibrium: illustration

Matching pennies — Player 2

		H (q)	T ($1-q$)
Player 1	H (r)	-1 , 1	1 , -1
	T ($1-r$)	1 , -1	-1 , 1

- Player 2选择 Head 和Tail无差异.
  - $EU_2(H, (r, 1-r)) = r \times 1+(1-r)\times(-1) =2r-1$
  - $EU_2(T, (r, 1-r)) = r\times(-1)+(1-r)\times 1 = 1-2r$
  - $EU_2(H, (r, 1-r)) = EU_2(T, (r, 1-r))$
    $2r-1= 1-2r$
    $4r = 2$ 从而 $r = 1/2$

- 所以, 根据定理2，((0.5, 0.5), (0.5, 0.5))是一个混合策略纳什均衡.

Use Theorem 2 to find mixed strategy Nash equilibrium: illustration

Employee Monitoring

		Manager	
		Monitor (q)	Not Monitor ($1-q$)
Employee	Work (r)	50 , 90	50 , 100
	Shirk ($1-r$)	0 , -10	100 , -100

- Employee选择 "Work"的期望收益
  - $EU_1(\text{Work}, (q, 1-q)) = q \times 50 + (1-q) \times 50 = 50$
- Employee选择 "Shirk"的期望收益
  - $EU_1(\text{Shirk}, (q, 1-q)) = q \times 0 + (1-q) \times 100 = 100(1-q)$
- Employee选择 "Work" 和 "Shirk" 无差异.
  - $50 = 100(1-q)$
  - $q = 1/2$

Use Theorem 2 to find mixed strategy Nash equilibrium: illustration

Employee Monitoring

		Manager: Monitor (q)	Manager: Not Monitor ($1-q$)
Employee	Work (r)	50 , 90	50 , 100
	Shirk ($1-r$)	0 , -10	100 , -100

- Manager选择“Monitor”的期望收益
  - $EU_2(\text{Monitor}, (r, 1-r)) = r\times 90+(1-r)\times(-10) =100r-10$
- Manager选择“Not”的期望收益
  - $EU_2(\text{Not}, (r, 1-r)) = r\times 100+(1-r)\times(-100) =200r-100$
- Manager选择“Monitor”和“Not”无差异
  $100r-10 =200r-100$ implies that $r=0.9$.
- 所以, 根据定理2，((0.9, 0.1), (0.5, 0.5))是一个混合策略纳什均衡.

Use Theorem 2 to find mixed strategy Nash equilibrium: illustration

Battle of sexes

		Pat	
		Opera (*q*)	Prize Fight (1-*q*)
Chris	Opera (*r*)	2 , 1	0 , 0
	Prize Fight (1-*r*)	0 , 0	1 , 2

- 使用定理2检查 ((2/3, 1/3), (1/3, 2/3)) 是否是一个混合策略纳什均衡？

Use Theorem 2 to find mixed strategy Nash equilibrium: illustration

Battle of sexes

Pat

		Opera (q)	Prize Fight ($1-q$)
Chris	Opera (r)	2 , 1	0 , 0
	Prize Fight ($1-r$)	0 , 0	1 , 2

- Chris选择Opera的期望收益
  - $EU_1(O, (q, 1-q)) = q \times 2 + (1-q) \times 0 = 2q$
- Chris选择Prize Fight的期望收益
  - $EU_1(F, (q, 1-q)) = q \times 0 + (1-q) \times 1 = 1-q$
- Chris选择Opera和Prize无差异
  - $EU_1(O, (q, 1-q)) = EU_1(F, (q, 1-q))$

    $2q = 1-q$

    $3q = 1$ 从而$q = 1/3$

Use Theorem 2 to find mixed strategy Nash equilibrium: illustration

Battle of sexes

		Pat: Opera (q)	Pat: Prize Fight ($1-q$)
Chris	Opera (r)	2 , 1	0 , 0
	Prize Fight ($1-r$)	0 , 0	1 , 2

- Pat选择Opera的期望收益
  - $EU_2(O, (r, 1-r)) = r \times 1 + (1-r) \times 0 = r$
- Pat选择Prize Fight的期望收益
  - $EU_2(F, (r, 1-r)) = r \times 0 + (1-r) \times 2 = 2 - 2r$
- Pat选择Opera和Prize无差异
  - $EU_2(O, (r, 1-r)) = EU_2(F, (r, 1-r))$

    $r = 2 - 2r$

    $3r = 2$ 从而 $r = 2/3$

Use Theorem 2 to find mixed strategy Nash equilibrium: illustration

Battle of sexes

		Pat	
		Opera (*q*)	Prize Fight (1-*q*)
Chris	Opera (*r*)	2 , 1	0 , 0
	Prize Fight (1-*r*)	0 , 0	1 , 2

- 所以, ((2/3, 1/3), (1/3, 2/3)) 是一个混合策略纳什均衡. 即,
- Chris以2/3的概率选择 Opera， 以1/3的概率选择 Prize Fight.
- Pat以1/3的概率选择 Opera， 以2/3的概率选择 Prize Fight.

Example 1

- Bruce 和Sheila要决定是去看歌剧还是去看职业摔跤表演.
- Sheila去看歌剧和职业摔跤分别可以得到效用4和1.
- Bruce去看歌剧和职业摔跤分别可以得到效用1和4.
- 他们同意使用以下方法决定去哪里:
  - Bruce和Sheila每人把一枚硬币放在咖啡桌上电视遥控器下面（假设他们不作弊看对方的硬币）. 他们数到3，同时显示他们的硬币. 如果他们的硬币显示一致 (都是heads,或都是tails), 那么Sheila决定去看歌剧还是职业摔跤, 而如果他们的硬币显示不一致 (heads, tails 或tails, heads)，那么 Bruce决定去哪里.

Example 1

		Sheila	
		H (q)	T ($1-q$)
Bruce	H (r)	1 , 4	4 , 1
	T ($1-r$)	4 , 1	1 , 4

- Bruce选Head的期望收益
  - $EU_1(H, (q, 1-q)) = q \times 1 + (1-q) \times 4 = 4-3q$
- Bruce选Tail的期望收益
  - $EU_1(T, (q, 1-q)) = q \times 4 + (1-q) \times 1 = 1+3q$
- Bruce选Head和Tail无差异
  - $EU_1(H, (q, 1-q)) = EU_1(T, (q, 1-q))$
    $4-3q = 1+3q$
    $6q = 3$ 从而 $q = 1/2$

Example 1

		Sheila H (q)	Sheila T ($1-q$)
Bruce	H (r)	1 , 4	4 , 1
Bruce	T ($1-r$)	4 , 1	1 , 4

- Sheila选Head的期望收益
  - $EU_2(H, (r, 1-r)) = r \times 4 + (1-r) \times 1 = 3r + 1$
- Sheila选Tail的期望收益
  - $EU_2(T, (r, 1-r)) = r \times 1 + (1-r) \times 4 = 4 - 3r$
- Sheila选Head和Tail无差异
  - $EU_2(H, (r, 1-r)) = EU_2(T, (r, 1-r))$
    $3r + 1 = 4 - 3r$
    $6r = 3$ 从而 $r = ½$
- ((1/2, 1/2), (1/2, 1/2)) 是一个混合策略纳什均衡.

Example 2

		L (q)	R ($1-q$)
Player 1	T (r)	<u>6</u> , 0	0 , <u>6</u>
	B ($1-r$)	3 , <u>2</u>	<u>6</u> , 0

Player 2

- Player 1选择T的期望收益
  - $EU_1(T, (q, 1-q)) = q \times 6 + (1-q) \times 0 = 6q$
- Player 1选择B的期望收益
  - $EU_1(B, (q, 1-q)) = q \times 3 + (1-q) \times 6 = 6-3q$
- Player 1选择T和B无差异
  - $EU_1(T, (q, 1-q)) = EU_1(B, (q, 1-q))$
    $6q = 6-3q$
    $9q = 6$ 从而 $q = 2/3$

Example 2

			Player 2	
			L (q)	R ($1-q$)
Player 1	T (r)		$\underline{6}$, 0	0 , $\underline{6}$
	B ($1-r$)		3 , $\underline{2}$	$\underline{6}$, 0

- Player 2选择L的期望收益
  - $EU_2(L, (r, 1-r)) = r \times 0+(1-r)\times 2 =2- 2r$
- Player 2选择R的期望收益
  - $EU_2(R, (r, 1-r)) = r\times 6+(1-r)\times 0 = 6r$
- Player 2选择L和R无差异
  - $EU_2(L, (r, 1-r)) = EU_2(R, (r, 1-r))$

    $2- 2r = 6r$

    $8r = 2$ 从而 $r = 1/4$

- ((1/4, 3/4), (2/3, 1/3)) 是一个混合策略纳什均衡.

Example 3:Market entry game

- 两家企业, Firm 1 和 Firm 2, 必须同时决定是否让他们的一家饭店进入一家购物中心.
- 每个企业有两个策略: Enter, Not Enter
- 企业如果选择 “Not Enter”, 它获得的利润为 0
- 如果一家企业选 “Enter”而另一家企业选 “Not Enter”，那么选“Enter”的企业得到 $500K
- 如果两家企业都选 “Enter”， 那么它们都损失 $100K，因为需求是有限的

Example 3:Market entry game

		Firm 2	
		Enter （ *q* ）	Not Enter （ 1–*q* ）
Firm 1	Enter （ *r* ）	-100 ， -100	500 ， 0
	Not Enter (1–*r*)	0 ， 500	0 ， 0

- 你能找到几个纳什均衡?
- 两个纯策略纳什均衡
  (Not Enter, Enter) and (Enter, Not Enter)
- 一个混合策略纳什均衡
  ((5/6, 1/6), (5/6, 1/6))
  即 r=5/6， q=5/6

Example 4

		Player 2	
		L (*q*)	R (1–*q*)
Player 1	T (*r*)	1 , 1	1 , 2
	B (1–*r*)	2 , 3	0 , 1

- 你能找到几个纳什均衡?
- 两个纯策略纳什均衡
  (B, L) and (T, R)
- 一个混合策略纳什均衡
  ((2/3, 1/3), (1/2, 1/2))
  即 $r=2/3$， $q=1/2$

Example: Rock, paper and scissors

- 两个参与人同时宣称*Rock*, *Paper*或 *Scissors*.
- Paper 胜 (包住) rock
- Rock 胜 (撞钝) scissors
- Scissors 胜 (剪破) paper
- 获得胜利的参与人从对手那里得到$1
- 如果参与人获得平局则不会得到支付

Example: Rock, paper and scissors

		Player 2		
		Rock	Paper	Scissors
Player 1	Rock	0 , 0	-1 , 1	1 , -1
	Paper	1 , -1	0 , 0	-1 , 1
	Scissors	-1 , 1	1 , -1	0 , 0

- 你能猜到一个混合策略纳什均衡吗?

2-player each with a finite number of pure strategies

- 参与人集合: {Player 1, Player 2}
- 策略集:
  player 1: $S_1 = \{ s_{11}, s_{12}, ..., s_{1J} \}$
  player 2: $S_2 = \{ s_{21}, s_{22}, ..., s_{2K} \}$
- 收益函数:
  player 1: $u_1(s_{1j}, s_{2k})$
  player 2: $u_2(s_{1j}, s_{2k})$
  for $j = 1, 2, ..., J$ and $k = 1, 2, ..., K$

2-player each with a finite number of pure strategies

Player 2

Player 1

	s_{21} (p_{21})	s_{22} (p_{22})	…….	s_{2K} (p_{2K})
s_{11} (p_{11})	$u_2(s_{11}, s_{21})$ $u_1(s_{11}, s_{21})$	$u_2(s_{11}, s_{22})$ $u_1(s_{11}, s_{22})$	…….	$u_2(s_{11}, s_{2K})$ $u_1(s_{11}, s_{2K})$
s_{12} (p_{12})	$u_2(s_{12}, s_{21})$ $u_1(s_{12}, s_{21})$	$u_2(s_{12}, s_{22})$ $u_1(s_{12}, s_{22})$	…….	$u_2(s_{12}, s_{2K})$ $u_1(s_{12}, s_{2K})$
….	……	…….	…….	……
s_{1J} (p_{1J})	$u_2(s_{1J}, s_{21})$ $u_1(s_{1J}, s_{21})$	$u_2(s_{1J}, s_{22})$ $u_1(s_{1J}, s_{22})$	……	$u_2(s_{1J}, s_{2K})$ $u_1(s_{1J}, s_{2K})$

- Player 1的混合策略: $p_1=(p_{11}, p_{12}, ..., p_{1J})$
- Player 2的混合策略: $p_2=(p_{21}, p_{22}, ..., p_{2K})$

Expected payoffs: 2-player each with a finite number of pure strategies

- Player 1纯策略s_{11}的期望收益:

  $EU_1(s_{11}, p_2)=p_{21}\times u_1(s_{11}, s_{21})+p_{22}\times u_1(s_{11}, s_{22})+\ldots+p_{2k}\times u_1(s_{11}, s_{2k})+\ldots+p_{2K}\times u_1(s_{11}, s_{2K})$

- Player 1纯策略s_{12}的期望收益:

  $EU_1(s_{12}, p_2)=p_{21}\times u_1(s_{12}, s_{21})+p_{22}\times u_1(s_{12}, s_{22})+\ldots+p_{2k}\times u_1(s_{12}, s_{2k})+\ldots+p_{2K}\times u_1(s_{12}, s_{2K})$

- ....
- Player 1纯策略s_{1J}的期望收益:

  $EU_1(s_{1J}, p_2)=p_{21}\times u_1(s_{1J}, s_{21})+p_{22}\times u_1(s_{1J}, s_{22})+\ldots+p_{2k}\times u_1(s_{1J}, s_{2k})+\ldots+p_{2K}\times u_1(s_{1J}, s_{2K})$

- Player 1混合策略p_1的期望收益:

$$v_1(p_1, p_2)=p_{11}\times EU_1(s_{11}, p_2)+p_{12}\times EU_1(s_{12}, p_2)+\ldots+p_{1j}\times EU_1(s_{1j}, p_2)+\ldots +p_{1J}\times EU_1(s_{1J}, p_2)$$

Expected payoffs: 2-player each with a finite number of pure strategies

- Player 2纯策略s_{21}的期望收益:

  $EU_2(s_{21}, p_1)=p_{11}\times u_2(s_{11}, s_{21})+p_{12}\times u_2(s_{12}, s_{21})+...+p_{1j}\times u_2(s_{1j}, s_{21})+...+p_{1J}\times u_2(s_{1J}, s_{21})$

- Player 2纯策略s_{22}的期望收益:

  $EU_2(s_{22}, p_1)=p_{11}\times u_2(s_{11}, s_{22})+p_{12}\times u_2(s_{12}, s_{22})+...+p_{1j}\times u_2(s_{1j}, s_{22})+...+p_{1J}\times u_2(s_{1J}, s_{22})$

- ....
- Player 2纯策略s_{2K}的期望收益:

  $EU_2(s_{2K}, p_1)=p_{11}\times u_2(s_{11}, s_{2K})+p_{12}\times u_2(s_{12}, s_{2K})+...+p_{1j}\times u_2(s_{1j}, s_{2K})+...+p_{1J}\times u_2(s_{1J}, s_{2K})$

- Player 2混合策略p_2的期望收益:

$$v_2(p_1,\ p_2)=p_{21}\times EU_2(s_{21}, p_1)+p_{22}\times EU_2(s_{22}, p_1)+...+p_{2k}\times EU_2(s_{2k}, p_1)+....+p_{2K}\times EU_2(s_{2K}, p_1)$$

Mixed strategy Nash equilibrium: 2-player each with a finite number of pure strategies

- 一个混合策略组合 $(p_1^*,\ p_2^*)$, 其中

  $$p_1^*=(p_{11}^*, p_{12}^*, \ldots, p_{1J}^*)$$
  $$p_2^*=(p_{21}^*, p_{22}^*, \ldots, p_{2K}^*)$$

  是一个混合策略均衡，如果player 1的混合策略p_1^*是对player 2的混合策略p_2^*的最优反应,同时p_2^*也是p_1^*的最优反应.
- 或者,对于player 1的所有混合策略p_1, $v_1(p_1^*,\ p_2^*) \geq v_1(p_1, p_2^*)$, 对于player 2的所有混合策略$p_2$, $v_2(p_1^*,\ p_2^*) \geq v_2(p_1^*, p_2)$.
- 即,给定 player 2的混合策略p_2^*, player 1如果偏离了p_1^*，那么她的境况将不会得到改善. 给定player 1的混合策略p_1^*, player 2如果偏离了 p_2^*，那么她的境况将不会得到改善.

2-player each with a finite number of pure strategies

- 定理 3 (纳什均衡的性质)
  - 一个混合策略组合 $(p_1^*,\ p_2^*)$, 其中

$$p_1^*=(p_{11}^*, p_{12}^*, ..., p_{1J}^*)$$
$$p_2^*=(p_{21}^*, p_{22}^*, ..., p_{2K}^*)$$

  是一个混合策略纳什均衡，当且仅当

  $v_1(p_1^*,\ p_2^*) \geq EU_1(s_{1j}, p_2^*)$, for $j = 1, 2, ..., J$

  $v_2(p_1^*,\ p_2^*) \geq EU_2(s_{2k}, p_1^*)$, for $k= 1, 2, ..., K$

2-player each with a finite number of pure strategies

- 定理 4 一个混合策略组合 (p_1^*, p_2^*), 其中

  $p_1^*=(p_{11}^*, p_{12}^*, \ldots, p_{1J}^*)$

  $p_2^*=(p_{21}^*, p_{22}^*, \ldots, p_{2K}^*)$

  是一个混合策略纳什均衡，当且仅当它们满足以下条件:
  - player 1: 对任何 m 和 n, 如果 $p_{1m}^*>0$， $p_{1n}^*>0$ 那么 $EU_1(s_{1m}, p_2^*) = EU_1(s_{1n}, p_2^*)$; 如果 $p_{1m}^*>0$， $p_{1n}^*=0$ 那么 $EU_1(s_{1m}, p_2^*) \geq EU_1(s_{1n}, p_2^*)$
  - player 2: 对任何 i 和 k, 如果 $p_{2i}^*>0$ and $p_{2k}^*>0$ 那么 $EU_2(s_{2i}, p_1^*) = EU_2(s_{2k}, p_1^*)$; 如果 $p_{2i}^*>0$ and $p_{2k}^*=0$ 那么 $EU_2(s_{2i}, p_1^*) \geq EU_2(s_{2k}, p_1^*)$

2-player each with a finite number of pure strategies

- 定理4告诉了我们什么?
  - 一个混合策略组合 (p_1^*, p_2^*), 其中 $p_1^*=(p_{11}^*, p_{12}^*, ..., p_{1J}^*)$, $p_2^*=(p_{21}^*, p_{22}^*, ..., p_{2K}^*)$ 是一个混合策略纳什均衡，当且仅当它们满足以下条件:
  - 给定 player 2的 p_2^*, player 1指定为正概率的每个纯策略的期望收益都相等, 且player 1指定为正概率的任何纯策略的期望收益都不会小于她指定为零概率的纯策略的期望收益.
  - 给定player 1的 p_1^*, player 2指定为正概率的每个纯策略的期望收益都相等,且player 2指定为正概率的任何纯策略的期望收益都不会小于她指定为零概率的纯策略的期望收益.

2-player each with a finite number of pure strategies

- 定理4意味着在以下情形中我们有混合策略纳什均衡
  - 给定player 2的混合策略, Player 1指定为正概率的纯策略之间是无差异的. 她指定为正概率的任何纯策略的期望收益都不会小于她指定为零概率的纯策略的期望收益.
  - 给定player 1的混合策略, Player 2指定为正概率的纯策略之间是无差异的. 她指定为正概率的任何纯策略的期望收益都不会小于她指定为零概率的纯策略的期望收益.

Theorem 4: illustration

Player 2

		L (0)	C (1/3)	R (2/3)
	T (3/4)	0 , 2	3 , 3	1 , 1
Player 1	M (0)	4 , 0	0 , 4	2 , 3
	B (1/4)	3 , 4	5 , 1	0 , 7

- 检查是否 ((3/4, 0, 1/4), (0, 1/3, 2/3)) 是一个混合策略纳什均衡
- Player 1:
  - $EU_1(T, p_2) = 0\times0+3\times(1/3)+1\times(2/3)=5/3$,
    $EU_1(M, p_2) = 4\times0+0\times(1/3)+2\times(2/3)=4/3$
    $EU_1(B, p_2) = 3\times0+5\times(1/3)+0\times(2/3)=5/3$.
  - Hence, $EU_1(T, p_2) = EU_1(B, p_2) > EU_1(M, p_2)$

Theorem 4: illustration

			Player 2	
		L (0)	C (1/3)	R (2/3)
	T (3/4)	0 , 2	3 , 3	1 , 1
Player 1	M (0)	4 , 0	0 , 4	2 , 3
	B (1/4)	3 , 4	5 , 1	0 , 7

- Player 2:
  - $EU_2(L, p_1)=2\times(3/4) + 0\times0 + 4\times(1/4)=5/2$,
    $EU_2(C, p_1)=3\times(3/4) + 4\times0 + 1\times(1/4)=5/2$,
    $EU_2(R, p_1)=1\times(3/4) + 3\times0 + 7\times(1/4)=5/2$.
  - Hence, $EU_2(C, p_1)=EU_2(R, p_1)\geq EU_2(L, p_1)$
- 所以,根据定理4， ((3/4, 0, 1/4), (0, 1/3, 2/3)) 是一个混合策略纳什均衡.

Example: Rock, paper and scissors

		Player 2		
		Rock (p_{21})	Paper (p_{22})	Scissors (p_{23})
Player 1	Rock (p_{11})	0 , 0	-1 , <u>1</u>	<u>1</u> , -1
	Paper (p_{12})	<u>1</u> , -1	0 , 0	-1 , <u>1</u>
	Scissors (p_{13})	-1 , <u>1</u>	<u>1</u> , -1	0 , 0

- 检查在$p_{11}>0$, $p_{12}>0$, $p_{13}>0$, $p_{21}>0$, $p_{22}>0$, $p_{23}>0$中是否存在一个混合策略纳什均衡.

Example: Rock, paper and scissors

		Player 2		
		Rock (p_{21})	Paper (p_{22})	Scissors (p_{23})
Player 1	Rock (p_{11})	0 , 0	-1 , 1	1 , -1
	Paper (p_{12})	1 , -1	0 , 0	-1 , 1
	Scissors (p_{13})	-1 , 1	1 , -1	0 , 0

- 如果每个参与人为她的每个纯策略都指定正概率, 那么根据定理4,对于每个参与人来说，她的三个纯策略无差异.

Example: Rock, paper and scissors

		Player 2		
		Rock (p_{21})	Paper (p_{22})	Scissors (p_{23})
Player 1	Rock (p_{11})	0 , 0	-1 , 1	1 , -1
	Paper (p_{12})	1 , -1	0 , 0	-1 , 1
	Scissors (p_{13})	-1 , 1	1 , -1	0 , 0

- Player 1的三个纯策略对她来说无差异：

  $EU_1(Rock, p_2) = 0\times p_{21} + (-1)\times p_{22} + 1\times p_{23}$

  $EU_1(Paper, p_2) = 1\times p_{21} + 0\times p_{22} + (-1)\times p_{23}$

  $EU_1(Scissors, p_2) = (-1)\times p_{21} + 1\times p_{22} + 0\times p_{23}$

- $EU_1(Rock, p_2) = EU_1(Paper, p_2) = EU_1(Scissors, p_2)$
- 连同 $p_{21} + p_{22} + p_{23} = 1$, 我们有三个方程和三个未知数.

Example: Rock, paper and scissors

		Player 2		
		Rock (p_{21})	Paper (p_{22})	Scissors (p_{23})
Player 1	Rock (p_{11})	0 , 0	-1 , <u>1</u>	<u>1</u> , -1
	Paper (p_{12})	<u>1</u> , -1	0 , 0	-1 , <u>1</u>
	Scissors (p_{13})	-1 , <u>1</u>	<u>1</u> , -1	0 , 0

- $0\times p_{21}+(-1)\times p_{22}+1\times p_{23}= 1\times p_{21}+0\times p_{22}+(-1)\times p_{23}$
  $0\times p_{21}+(-1)\times p_{22}+1\times p_{23} = (-1)\times p_{21}+1\times p_{22}+0\times p_{23}$
  $p_{21}+ p_{22}+ p_{23}=1$
- 解得
  $p_{21}= p_{22}= p_{23}=1/3$

Example: Rock, paper and scissors

		Player 2		
		Rock (p_{21})	Paper (p_{22})	Scissors (p_{23})
Player 1	Rock (p_{11})	0 , 0	-1 , <u>1</u>	<u>1</u> , -1
	Paper (p_{12})	<u>1</u> , -1	0 , 0	-1 , <u>1</u>
	Scissors (p_{13})	-1 , <u>1</u>	<u>1</u> , -1	0 , 0

- Player 2的三个纯策略对她来说无差异：

  $\mathbf{EU_2(Rock,}\ p_1\mathbf{)=0}\times p_{11}\mathbf{+(-1)}\times p_{12}\mathbf{+1}\times p_{13}$

  $\mathbf{EU_2(Paper,}\ p_1\mathbf{)=1}\times p_{11}\mathbf{+0}\times p_{12}\mathbf{+(-1)}\times p_{13}$

  $\mathbf{EU_2(Scissors,}\ p_1\mathbf{)=(-1)}\times p_{11}\mathbf{+1}\times p_{12}\mathbf{+0}\times p_{13}$

- $\mathbf{EU_2(Rock,}\ p_1\mathbf{)= EU_2(Paper,}\ p_1\mathbf{)=EU_2(Scissors,}\ p_1\mathbf{)}$
- 连同 $p_{11}+p_{12}+p_{13}=1$,我们有三个方程和三个未知数.

Example: Rock, paper and scissors

		Player 2		
		Rock (p_{21})	Paper (p_{22})	Scissors (p_{23})
Player 1	Rock (p_{11})	0 , 0	-1 , <u>1</u>	<u>1</u> , -1
	Paper (p_{12})	<u>1</u> , -1	0 , 0	-1 , <u>1</u>
	Scissors (p_{13})	-1 , <u>1</u>	<u>1</u> , -1	0 , 0

- $0 \times p_{11} + (-1) \times p_{12} + 1 \times p_{13} = 1 \times p_{11} + 0 \times p_{12} + (-1) \times p_{13}$
  $0 \times p_{11} + (-1) \times p_{12} + 1 \times p_{13} = (-1) \times p_{11} + 1 \times p_{12} + 0 \times p_{13}$
  $p_{11} + p_{12} + p_{13} = 1$

- 解得
  $p_{11} = p_{12} = p_{13} = 1/3$

Example: Rock, paper and scissors

		Player 2		
		Rock (1/3)	Paper (1/3)	Scissors (1/3)
Player 1	Rock (1/3)	0 , 0	-1 , 1	1 , -1
	Paper (1/3)	1 , -1	0 , 0	-1 , 1
	Scissors (1/3)	-1 , 1	1 , -1	0 , 0

- Player 1: $EU_1(Rock, p_2) = 0\times(1/3)+(-1)\times(1/3)+1\times(1/3)=0$
  $EU_1(Paper, p_2) = 1\times(1/3)+0\times(1/3)+(-1)\times(1/3)=0$
  $EU_1(Scissors, p_2) = (-1)\times(1/3)+1\times(1/3)+0\times(1/3)=0$
- Player 2: $EU_2(Rock, p_1)=0\times(1/3)+(-1)\times(1/3)+1\times(1/3)=0$
  $EU_2(Paper, p_1)=1\times(1/3)+0\times(1/3)+(-1)\times(1/3)=0$
  $EU_2(Scissors, p_1)=(-1)\times(1/3)+1\times(1/3)+0\times(1/3)=0$
- 所以,根据定理4， (p_1=(1/3, 1/3, 1/3), p_2=(1/3, 1/3, 1/3))是一个混合策略纳什均衡.

Example: Rock, paper and scissors

		Player 2		
		Rock (p_{21})	Paper (p_{22})	Scissors (p_{23})
Player 1	Rock (p_{11})	0 , 0	-1 , <u>1</u>	<u>1</u> , -1
	Paper (p_{12})	<u>1</u> , -1	0 , 0	-1 , <u>1</u>
	Scissors (p_{13})	-1 , <u>1</u>	<u>1</u> , -1	0 , 0

- 检查是否存在这样一个混合策略纳什均衡，其中， p_{11}, p_{12}, p_{13}中有一个为正值，且p_{21}, p_{22}, p_{23}中至少有两个为正值.
- 答案是*不存在*.

Example: Rock, paper and scissors

		Player 2		
		Rock (p_{21})	Paper (p_{22})	Scissors (p_{23})
Player 1	Rock (p_{11})	0 , 0	-1 , 1	1 , -1
	Paper (p_{12})	1 , -1	0 , 0	-1 , 1
	Scissors (p_{13})	-1 , 1	1 , -1	0 , 0

- 检查是否存在这样一个混合策略纳什均衡，其中， p_{11}, p_{12}, p_{13}中有两个为正值，且p_{21}, p_{22}, p_{23}中至少有两个为正值.
- 答案是*不存在*.

Example: Rock, paper and scissors

		Player 2		
		Rock (p_{21})	Paper (p_{22})	Scissors (p_{23})
Player 1	Rock (p_{11})	0 , 0	-1 , <u>1</u>	<u>1</u> , -1
	Paper (p_{12})	<u>1</u> , -1	0 , 0	-1 , <u>1</u>
	Scissors (p_{13})	-1 , <u>1</u>	<u>1</u> , -1	0 , 0

- 所以, 根据定理4，(p_1=(1/3, 1/3, 1/3), p_2=(1/3, 1/3, 1/3)) 是惟一的混合策略纳什均衡.

Nash Theorem

- 定理 (纳什, 1950): 在n个参与者的标准式博弈G = {S_1,…, S_n; u_1,…, u_n}中，如果n是有限的，且对每个i，S_i是有限的，则博弈存在至少一个纳什均衡，均衡可能包括混合均衡。
- 纳什均衡的存在性！
- 定理中的假定是均衡存在性的充分条件，却不是必要条件——还有许多博弈，虽不满足定理假定的条件，却同样存在一个或多个纳什均衡。

习题1.5

- 思路：对任意一个参与人来说（以参与人1为例），加入q’之前，$q_m/2$是q_c的严格劣势策略，加入q’后，不存在严格劣势战略，则新的策略组合(q_c, q’)和($q_m/2$, q_c)中参与人1的收益相等。得q’=5(a-c)/12。

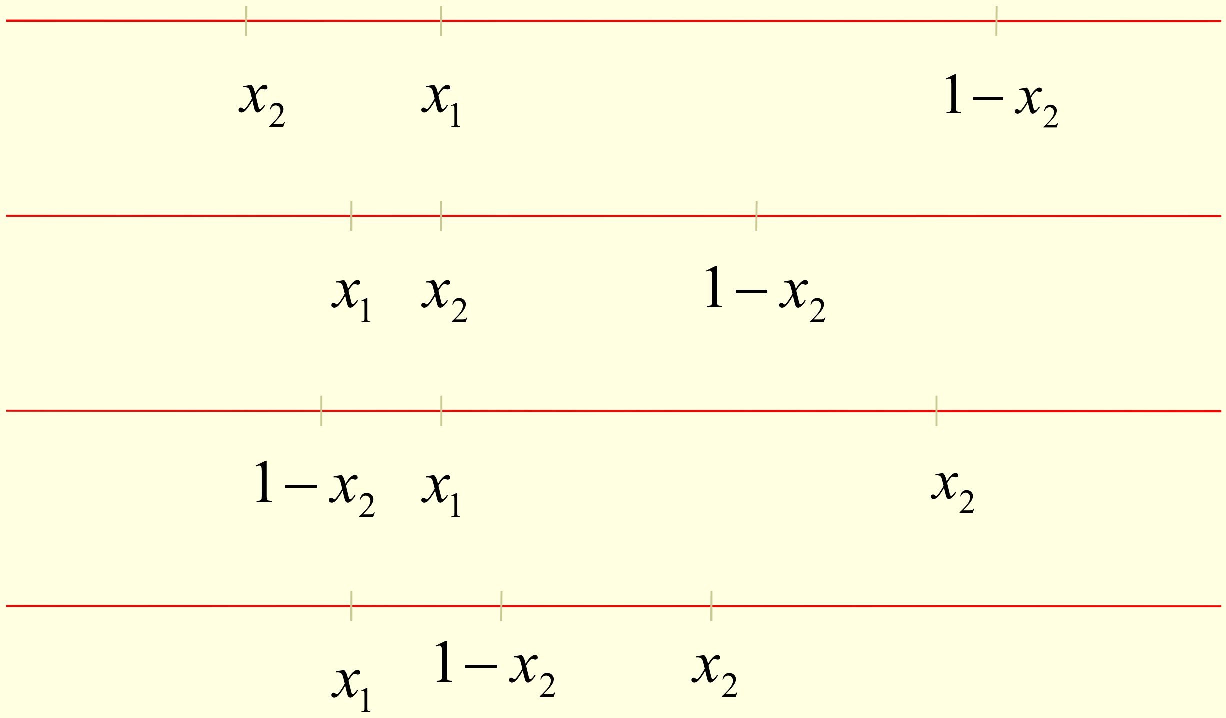
Player 2

Player 1		$q_m/2$	q_c	q’
	$q_m/2$	A/8， A/8	5A/48， 5A/36	A/12， 5A/36
	q_c	5A/36，5A/48	A/9， A/9	A/12， 5A/48
	q’	5A/36， A/12	5A/48， A/12	5A/72， 5A/72

- 在得到的三个纳什均衡中，只有(q_c, q_c)中每个企业的福利A/9，A/9都比他们互相合作时($q_m/2$, $q_m/2$)的福利A/8，A/8要低，其中A=(a-c)^2。

习题1.8

- 在Hotelling模型中，如果选民是均匀分布在[0, 1]之间的，则更靠近中间1/2位置处的竞选人会赢得选举。



习题1.8

- 参与人集合：{竞选人1}，{竞选人2}
- 策略集：$S_1=\{x_1\}=[0,1]$，$S_2=\{x_2\}=[0,1]$
- 收益函数：w表示竞选成功，a表示不分胜负，0表示竞选失败

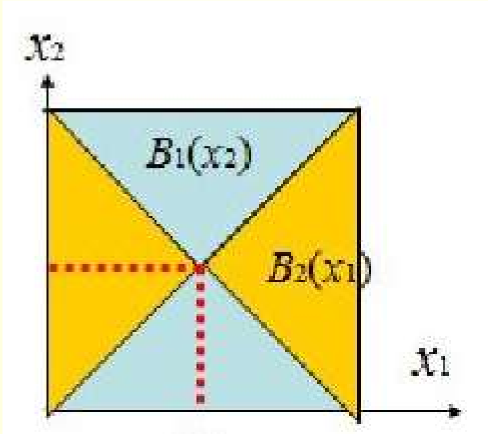
$$U_1=\begin{cases}0, & if\ 0\le x_1<x_2<1-x_1\le 1 \\ & or\ 0\le 1-x_1<x_2<x_1\le 1 \\ a, & if\ 0\le x_1=x_2\le 1 \\ w, & if\ 0\le x_2<x_1<1-x_2\le 1 \\ & or\ 0\le 1-x_2<x_1<x_2\le 1\end{cases}\qquad U_2=\begin{cases}0, & if\ 0\le x_2<x_1<1-x_2\le 1 \\ & or\ 0\le 1-x_2<x_1<x_2\le 1 \\ a, & if\ 0\le x_1=x_2\le 1 \\ w, & if\ 0\le x_1<x_2<1-x_1\le 1 \\ & or\ 0\le 1-x_1<x_2<x_1\le 1\end{cases}$$

习题1.8

- 最优反应对应的交点在 $x_1 = x_2 = 0.5$ 位置处。

$$B_1(x_2) = \begin{cases} x_1 \in \{x_1 : x_2 < x_1 < 1 - x_2\} & x_2 < 0.5 \\ x_1 = x_2 & x_2 = 0.5 \\ x_1 \in \{x_1 : 1 - x_2 < x_1 < x_2\} & x_2 > 0.5 \end{cases}$$

$$B_2(x_1) = \begin{cases} x_2 \in \{x_2 : x_1 < x_2 < 1 - x_1\} & x_1 < 0.5 \\ x_1 = x_2 & x_1 = 0.5 \\ x_2 \in \{x_2 : 1 - x_1 < x_2 < x_1\} & x_1 > 0.5 \end{cases}$$



习题1.8

- 两竞选人的Hotelling模型的纳什均衡是$x_1 = x_2 = 0.5$。
- 三竞选人的Hotelling模型是否存在纯策略纳什均衡?
- 考虑以下可能的情况:
  - 三个竞选人在同一位置;
  - 三个竞选人中，有两个在同一位置;
  - 三个竞选人在三个不同的位置。