Advanced Macroeconomics
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Midterm Exam I (Open-Book)
Undergraduate Economics Program, HUST
Thursday, April/12/2016

Name: ________________________ **Student ID:**________________________

1. (50 points) Consider an economy depicted by a Solow growth model with a **constant returns to scale** and **Cobb-Douglas** production function

$$F\left(A_K(t)K(t), A_L(t)L(t)\right) = \left[A_K(t)K(t)\right]^{\alpha}\left[A_L(t)L(t)\right]^{1-\alpha}, \tag{1}$$

where $\alpha \in (0,1)$, $\frac{\dot{A}_K(t)}{A_K(t)} \equiv g_K > 0$, $\frac{\dot{A}_L(t)}{A_L(t)} \equiv g_L > 0$, and $\frac{\dot{L}(t)}{L(t)} \equiv 0$. The saving rate is s and depreciation rate of capital equals δ. Prove that there exists a **globally stable steady state** for this economy.

2. (50 points) Consider an OLG growth model with all settings identical to those in the textbook except the life-time utility function (2.43) now becomes

$$U_t = C_{1t}^{\eta} C_{2t+1}^{1-\eta}, \tag{2}$$

with $\eta \in (0,1)$. The production function takes form $f(k) = k^{\alpha}$. Characterize the dynamic equilibrium of the economy.

1. $F = [A_K(t) K(t)]^{\alpha} [A_L(t) L(t)]^{1-\alpha}$

$= [K(t)]^{\alpha} [([A_K(t)]^{\alpha/(1-\alpha)} A_L(t)) L(t)]^{1-\alpha}$

def $A(t) = [A_K(t)]^{\alpha/1-\alpha} A_L(t)$ $\Rightarrow F = K^{\alpha}(AL)^{1-\alpha}$.

$k(t) = \frac{K(t)}{A(t) L(t)}$

$$\frac{\dot{k}(t)}{k(t)} = \frac{sF - \delta K(t)}{K(t)} - \frac{\dot{A}(t)}{A(t)} - \frac{\dot{L}(t)}{L(t)} = 0$$

$$= s[k(t)]^{\alpha-1} - \delta - \frac{\alpha}{1-\alpha} g_K - g_L.$$

Steady-state $\dot{k}(t) = 0. \Rightarrow k^* = \left(\frac{s}{\delta + g_L + \frac{\alpha}{1-\alpha} g_K}\right)^{\frac{1}{\alpha-1}}$

在BGP上. $k(t)$ 增速为0.

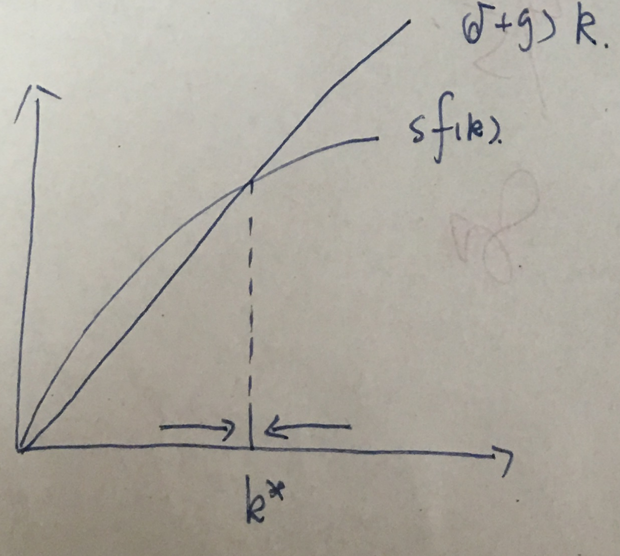
$K(t)/L(t)$ 增速等于 $\frac{\dot{A}(t)}{A(t)} = \frac{\alpha}{1-\alpha} g_K + g_L = g$.

人均产出增速 $g = \frac{\alpha}{1-\alpha} g_K + g_L$.

Proof 初始状态为任意 $k(0)$ 经济会收敛于 k^*.

当 $k(0) < k^*$ $\dot{k}(t) > 0$. $k(t) \to k^*$

当 $k(0) > k^*$ $\dot{k}(t) < 0$. $k(t) \to k^*$



α. Household behavior.

① $u_t = C_{1t}^{\eta} C_{2t+1}^{1-\eta}$.

s.t. $C_{1t} + \frac{1}{1+r_{t+1}} C_{2t+1} = A_t w_t$.

解之. $L = C_{1t}^{\eta} C_{2t+1}^{1-\eta} - \lambda\left(C_{1t} + \frac{C_{2t+1}}{1+r_{t+1}} - A_t w_t\right)$

F.O.C $\quad \eta C_{1t}^{\eta-1} C_{2t+1}^{1-\eta} - \lambda = 0.$ ①

$(1-\eta) C_{1t}^{\eta} C_{2t+1}^{-\eta} - \frac{\lambda}{1+r_{t+1}} = 0.$ ②

$C_{1t} + \frac{1}{1+r_{t+1}} C_{2t+1} = A_t w_t.$ ③

由①.②. $\frac{C_{1t}}{C_{2t+1}} = \frac{\eta}{(1-\eta)(1+r_{t+1})}$.

$\Rightarrow \begin{cases} C_{1t} = \eta A_t w_t. \\ C_{2t+1} = (1-\eta)(1+r_{t+1}) A_t w_t. \end{cases} \Rightarrow s = 1-\eta.$

②. 两期消费边际效用相等. 1期减少 ΔC. 2期增加 $(1+r_{t+1})\Delta C$.

$\eta C_{1t}^{\eta-1} C_{2t+1}^{1-\eta} \Delta C = (1-\eta) C_{1t}^{\eta} C_{2t+1}^{-\eta} (1+r_{t+1}) \Delta C.$

$\Rightarrow \frac{C_{1t}}{C_{2t+1}} = \frac{\eta}{(1-\eta)(1+r_{t+1})}$

Equation of Motion of k.

$K_{t+1} = s(r_{t+1}) L_t A_t w_t.$

$\Rightarrow k_{t+1} = \frac{1}{(1+n)(1+g)} s(r_{t+1}) w_t.$

$= \frac{1}{(1+n)(1+g)} s[f'(k_{t+1})][f(k_t) - k_t f'(k_t)].$

$= \frac{1}{(1+n)(1+g)} (1-\eta)(k_t^{\alpha} - \alpha k_t^{\alpha}) = \frac{(1-\eta)(1-\alpha)}{(1+n)(1+g)} k_t^{\alpha}.$

Steady state. $k_{t+1} = k_t$.

$\Rightarrow k^* = \frac{(1-\eta)(1-\alpha)}{(1+n)(1+g)} (k^*)^{\alpha}.$

$\Rightarrow k^* = \left[\frac{(1-\eta)(1-\alpha)}{(1+n)(1+g)}\right]^{\frac{1}{1-\alpha}}$

$y^* = (k^*)^{\alpha} = \left[\frac{(1-\eta)(1-\alpha)}{(1+n)(1+g)}\right]^{\frac{\alpha}{1-\alpha}}.$