

**Final Exam (Open-Book)**  
**Advanced Macroeconomics**  
**Instructed by Xu & Yi**  
**Undergraduate Program in Economics, HUST**  
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1. ( $10' + 10' + 15' + 15' = 50$  points)

- (a) There are typos (印刷错误) within Equations (6.38) and (6.39) in your textbook. Point them out and correct the equations.
- (b) It is said that if the menu cost is higher than some threshold level, then it is a Nash Equilibrium for all firms not to adjust their prices given a shock in monetary supply. How is the threshold defined in your textbook?
- (c) A firm is interested in finding out its *relative price*  $r_i$ , to figure out the optimal production plan. However, it can never observe its relative price perfectly. Instead, it builds a *prior distribution* about  $r_i$ , and observes a signal as a realization of a random variable  $p_i = r_i + p$  to form a *posterior distribution* of  $r_i$  using *Bayesian updating*. Specifically, the prior of  $r_i$  is the normal distribution  $\mathcal{N}(\alpha, \frac{1}{A})$ , and the firm knows  $p \sim \mathcal{N}(\beta, \frac{1}{B})$ . What is the *posterior distribution* of  $r_i$  given the prior distribution and signal  $p_i$  observed as described above?
- (d) According to the explanations in your textbook, which parameter will be influenced by frequently implemented monetary policies? How is it related to the famous “Lucas Critique”?

2. (10' + 10' + 15' + 15' = 50 points) **Calvo Model with Heterogeneous Firms.**

Consider a revised version of the Calvo model: We divide all firms into two subsets, group A and group B. Proportion  $\lambda$  of the firms belong to group A and the remaining proportion  $(1-\lambda)$  of firms belong to group B, where  $\lambda \in [0, 1]$ . Each firm in the group A gets the chance to change its price in each stage with probability  $\alpha$ , and the probability for a firm in the group B to do so is  $\gamma$ .  $0 < \alpha \leq \gamma < 1$ . All other settings remain the same as in your textbook.

- (a) Give the revised version of equation (7.54). (Hint: you need to distinguish between  $\chi_t^A$  and  $\chi_t^B$ .)
- (b) Define  $\pi_t^A = \chi_t^A - p_{t-1}$  and  $\pi_t^B = \chi_t^B - p_{t-1}$ . Use the two equations to rephrase your answer in (a). What economic intuition does your result tell?
- (c) Give the revised version of equation (7.60). You should get your result in the form of  $\pi_t = \kappa y_t + \xi_A E_t \pi_{t+1}^A + \xi_B E_t \pi_{t+1}^B$ , where  $\kappa$ ,  $\xi_A$ , and  $\xi_B$  are constant parameters.
- (d) Given  $\gamma > \alpha$ ,  $\frac{\partial \kappa}{\partial \lambda} \stackrel{\leq}{\geq} 0$ ? What is the economic intuition of your result?

## Solution Hints

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Please do me a favor by pointing out the mistakes and typos, if any, that I have made.  
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1.(a) Correct ones should be:

$$U = C - \frac{1}{\gamma}L^\gamma, \gamma > 1, \quad \text{and} \quad C = \left[ \int_{i=1}^1 C_i^{(\eta-1)/\eta} di \right]^{\eta/(\eta-1)}, \eta > 1.$$

1.(b) As shown through equations (6.62)-(6.64) in your textbook. It is a Nash Equilibrium for all firms not to adjust their prices if the menu cost  $Z > \pi_{ADJ} - \pi_{FIXED}$ .

1.(c) The posterior distribution should be:

$$r_i|p_i \sim \mathcal{N}\left(\frac{A\alpha + B(p_i - \beta)}{A + B}, \frac{1}{A + B}\right)$$

1.(d) Frequently implemented monetary policies will make the overall price level more volatile, so it decreases  $B$ . As a result, other things equal, the firm gives less weight to signal  $p_i$  whiling making her optimal decision, i.e., after observing an increase in the price of its products, the firm is less (more) willing to believe that the higher price is companioned by a higher *relative price* (overall price level), which is crucial to make the production plan. Consequently, the monetary policies, which are supposed to take advantage of the positive relationship between price level and output, tend to diminish the strength of the relationship.

2.(a) Now equation (7.53) becomes:

$$p_t = \lambda\alpha\chi_t^A + \lambda(1 - \alpha)p_{t-1} + (1 - \lambda)\gamma\chi_t^B + (1 - \lambda)(1 - \gamma)p_{t-1}, \quad (1)$$

which in turn gives us

$$\begin{aligned}\pi_t &= p_t - p_{t-1} \\ &= \lambda\alpha(\chi_t^A - p_{t-1}) + (1 - \lambda)\gamma(\chi_t^B - p_{t-1}).\end{aligned}\quad (2)$$

2.(b) Equation (2) now becomes:

$$\pi_t = \lambda\alpha\pi_t^A + (1 - \lambda)\gamma\pi_t^B. \quad (3)$$

Intuition is pretty straightforward: Inflation at  $t$  is caused by the firms in Group A and Group B that successfully receive the opportunity to flexibly adjust their prices. For the former, their mass is  $\lambda\alpha$ ; for the latter, their mass is  $(1 - \lambda)\gamma$ .

2.(c) Note that the process throughout equations (7.55)-(7.58) still works here. And we do not need to distinguish between  $p_t^{A*}$  and  $p_t^{B*}$  because they are the same (recalling the initial definition of  $p_t^*$ ). What is more,  $p_t^* - p_t = \phi y_t$  still holds. Consequently, we have equations:

$$\pi_t^A - \pi_t = (\chi_t^A - p_{t-1}) + (p_t - p_{t-1}) = [1 - \beta(1 - \alpha)]\phi y_t + \beta(1 - \alpha)E_t\pi_{t+1}^A \quad (4)$$

$$\pi_t^B - \pi_t = (\chi_t^B - p_{t-1}) + (p_t - p_{t-1}) = [1 - \beta(1 - \gamma)]\phi y_t + \beta(1 - \gamma)E_t\pi_{t+1}^B \quad (5)$$

Combining equations (3), (4), and (5) together gives us

$$\begin{aligned}\pi_t &= \frac{\lambda\alpha[1 - \beta(1 - \alpha)] + \gamma(1 - \lambda)[1 - \beta(1 - \gamma)]}{1 - \lambda\alpha - \gamma + \lambda\gamma}\phi y_t + \frac{\lambda\alpha\beta(1 - \alpha)}{1 - \lambda\alpha - \gamma + \lambda\gamma}E_t\pi_{t+1}^A \\ &\quad + \frac{(1 - \lambda)\gamma\beta(1 - \gamma)}{1 - \lambda\alpha - \gamma + \lambda\gamma}E_t\pi_{t+1}^B\end{aligned}\quad (6)$$

In other words,  $\kappa = \frac{\lambda\alpha[1 - \beta(1 - \alpha)] + \gamma(1 - \lambda)[1 - \beta(1 - \gamma)]}{1 - \lambda\alpha - \gamma + \lambda\gamma}\phi$ ,  $\xi_A = \frac{\lambda\alpha\beta(1 - \alpha)}{1 - \lambda\alpha - \gamma + \lambda\gamma}$ , and  $\xi_B = \frac{(1 - \lambda)\gamma\beta(1 - \gamma)}{1 - \lambda\alpha - \gamma + \lambda\gamma}$ .

2.(d) With  $0 < \alpha < \gamma < 1$ , it is straightforward to see  $\partial\kappa/\partial\lambda < 0$  for  $\lambda \in (0, 1)$ .

Intuition: Type A firms adjust prices less frequently than type B firms. A greater  $\lambda$  means there are more type A firms, and thus implies a higher nominal rigidity, e.g., monetary shocks lead to smaller price changes and greater output changes.